ASECU South Eastern Europe Journal of Economics 2 (2004) 81-100 SEEJE

MODELLING RETURNS ON STOCK INDICES FOR WESTERN AND CENTRAL EUROPEAN STOCK EXCHANGES - A MARKOV SWITCHING APPROACH

JEDRZEJ BIALKOWSKI*

Abstract
In this paper a Markov switching mixture of normal distributions is applied to the monthly returns on the main stock indices for emerging financial markets in central Europe (BUX, PX50 and WIG). Additionally the results are compared to those obtained for western Europe (DAX, CAC40 and FTSE100). The results of model comparison suggest that the Markov switching mixture of normal distributions has substantially more descriptive validity than a single normal distribution. Nevertheless, there is no clear indication that, in modelling monthly returns, the Markov switching mixture of three normal distributions is superior to the mixture of two. Finally, the ability of the models to describe returns during international financial crises is evaluated.

JEL Classification: C22, E37, F36, G12

Keywords: Markov switching mixture of normal distributions, Modelling returns, Emerging stock markets in central Europe, International financial crises

1. Introduction

Over the past four decades researchers have been preoccupied with determining the distribution of market returns. The attributes of the distribution are a decisive assumption of portfolio analysis, theoretical models of capital asset prices, and deri-

---

* Postgraduate Research Programme ‘Capital Markets and Finance in the Enlarged Europe’ Department of Economics European University Viadrina, P.O. BOX 1786 D-15207 Frankfurt (Oder), Germany, e-mail: bialkowski@euv-frankfurt-o.de.

The author would like to thank Professor Kwan Choi, Professor Nicolas Sarantis, and other participants of the EEFS conference in Bologna 2003 for many useful suggestions on a previous version of this work. Also the author would like to thank the three anonymous referees for valuable comments on a previous draft of this paper. Of course, any remaining errors remain the responsibility the author.

vative valuation. From the point of view of financial theory, the most convenient assumption is to consider that a normal distribution with stationary parameters approximates a distribution of observed returns. However, daily returns from many assets, portfolios and main indices seem to be drawn from non-normal distribution. From the classic paper of Fama (1965) it is known that the empirical distribution of returns from stocks tends to display more kurtosis and have a pronounced higher peak than allowed under the normality hypothesis. However, according to the evidence provided by Blattberg and Gonedes (1965) monthly returns conform to the normality hypothesis.

The results mentioned were obtained for developed markets. Due to the fact that returns on assets from emerging markets are more volatile (see Bekaert and Harvey (2003), Harvey (1991) and (1995)), one can expect that it will be even more difficult to identify their distribution. A variety of approaches to modelling returns have been adopted. The work of Kon (1984) contains their review.

This paper is innovative insomuch as it introduces application of Markov switching distribution to modelling returns on indices from central and eastern European exchanges, namely the Budapest Stock Index (BUX), Prague Stock Exchange 50 Index (PX50) and Warsaw Stock Exchange Index (WIG). Additionally, results are compared with those obtained for indices from western Europe, like DAX, CAC40, FTSE100.

The idea of Markov switching models (here Markov switching mixture of normal distributions) consists of recognizing different states of financial markets. During some of them the market is much more volatile, and these are called crisis states. Thus, there should be a significant difference between distributions from which returns in a balanced market and in an unstable one are drawn. Turner et al. (1989) assume that each state is described by a normal distribution, but with different parameters. It is worthwhile to highlight the fact that three different methods of evaluation of the model selection are applied in this work. One of them is based on a likelihood ratio test. The second one determines whether the chosen model for a particular stock market index is constant over time. In other words, a Markov switching model is fitted to an increasing sample, and the likelihood ratio test is adjusted. Finally, the method of moment comparison is applied for verification of selection of the model.

The motivation to explore the properties of returns from European emerging markets comes from the increasing interest in the economies of central European countries which will soon become members of the EU. From a perspective of international investors who can benefit from investing in newly available emerging markets (see Köke (1999), Harvey (1995)), it is essential to learn more about the properties of returns observed on those markets. The attention of global market participants is attracted to emerging markets by their attributes like impressive expected returns, and the low correlations with developed countries' equity. However, there is a troublesome quality of these markets, namely the high volatility observed especially during a crisis (see Ang and Bekaert (2002), Harvey (1995)). Therefore the suitable model should make possible an analysis of returns properties during the calm and crisis periods. Undoubtedly the Markov switching framework gives such an opportunity. Especially since it was successfully applied to model returns on indices of well-developed capital markets. (among other Ceccetti (1990), Hamilton and Lin (1996), Rydén et al. (1998), Schaller and Norden (1997), Turner et al. (1989)). The motivation for applying this particular methodology comes also from the fact that in the existing literature there is only the paper of Linne (2002) in which Markov switching models for modelling returns of assets from Central and Eastern Europe are applied. In contrast, in a mentioned work a different model than the one presented here was considered.

Furthermore the question about the difference between leverage effect observed on mature and emerging markets in Europe, motivates this study. Black (1976) was probably the first who documented a negative relationship between current stock returns and future volatility, which he attributed to the leverage effect. Since his work the mentioned relationship has been the subject of numerous empirical studies (see among others Koutmmos and Saidi (1995), Engle and Ng (1993), Schwert (1990)). This paper extends the existing literature in two ways: first, it applies the Markov switching mixture of normal distributions to the examination of leverage effect; and second, its analysis covers the mature and emerging markets.

From a theoretical viewpoint, this examination seems interesting because it focuses, among other things, on the problem of finding a suitable model to measure the skewness and kurtosis of returns. There are at least a couple of papers which have examined the asset allocation problem with special attention paid to positive skewness (see Peiró (2002), Chunhchinda et al. (1997), Lai (1991)). The crucial issue for these studies is the selection of an adequate return distribution.

In addition the paper also poses several interesting questions for investors like: Is it possible to distinguish distinct states in stock markets returns? Do stock returns exhibit features which are common across European mature and emerging markets? Are the shifts in returns related to certain well identified crises?

The paper is organized as follows. Section 2 contains data description and summary statistics of the data. Section 3 provides a description of the model, methods of its selection and parameter estimation. The results of the model selection and estimation (with analysis of vulnerability of crises for different markets) are carried out in section 4. Finally, section 5 presents conclusions.

2. Data

The sample used for this study comprises monthly logarithmic returns measured in percentage terms for the CAC40, DAX, FTSE100, BUX, PX50, and WIG index for the period from January 1995 to November 2002. Daily quotations for these indices are available on the internet site of Parkiet,[1] which is a stock exchange magazine. The summary statistics of monthly returns for examined markets are presented in Table 1.

Table 1. Summary Statistics for Data

Moments	*DAX*	*CAC40*	*FTSE100*	*BUX*	*PX50*	*WIG*
Mean μ_1	0.287	0.468	0.214	1.443	-0.091	0.485
Variance σ_1^2	48.36	37.87	16.78	109.8	55.29	92.68
Skewness	-1.009	-0.393	-0.604	-0.676	-0.376	0.219
Excess kurtosis	1.980	0.052	0.502	2.959	0.922	2.415
Likelihood $N(\mu_1, \sigma_1^2)$	-318.53	-306.92	-268.26	-357.27	-324.89	-349.42

The distinction between statistical properties of monthly returns for western European markets and central European markets is easily recognizable. The average variance and excess kurtosis calculated for the BUX, PX50 and WIG is more than double that for the DAX, CAC40 and FTSE100. Furthermore, skewness is comparable among all indices. The coefficients of skewness for most indices are negative, whereas the Polish stock returns exhibit a positive value.

One question, which is fundamental when dealing with monthly returns is the issue of normality hypothesis Blattberg and Gonedes (1965). The measure of skewness and excess kurtosis shows that the distributions of the returns are leptokurtic relative to the normal distribution, with one exception: namely the CAC40. For this index excess kurtosis was equal to 0.052. However, formal tests[2] for normality cannot reject the null hypothesis for CAC40, FTSE100, BUX, PX50, WIG. In the case of the DAX normality is rejected.

1. Further information is available on web site www.parkiet.com.pl.
2. The following tests are applied: Shapiro-Wilk, Kolomogorov-Smirnov, Cramee-von Mises, Anderson-Darling.

3. Methodology

3.1 Model

The main effort of researchers, who deal with the problem of determining the suitable model of stock returns, is to construct the model which will be able to reproduce the attributes of stock returns - such as asymmetry and fat tails in comparison to single normal distribution. The Markov switching models belong to a general class of mixtures of distributions, which have the ability to flexibly approximate general classes of density functions. Moreover, those models are able to generate values for the skewness and excess kurtosis close to those observed for real market data (see Timmermann (2000)).

In this study the Markov switching mixture of normal distribution described by Hamilton (1990) is applied. It was successfully used by Turner et al. (1989) for modelling excess returns of the S&P 500 index. In contrast to both those papers, where a financial market could be only in two states, the case of three states is examined, too.

The model postulates the existence of an unobservable Markov chain S_t that can take one of K-values. Each of them is attainable for the process S_t. When the financial market is in one of the K-states then returns on the index have been drawn from normal distribution, which corresponds to this state. A further assumption is that evolution of Markov chain is described by transition matrix **P**.

In this paper the following notation has been adopted. The return in month t is equal to y_t. The total number of monthly observation for each index is equal to T. Finally, the Markov switching mixture of K-normal distributions MSMIXN(K) has the following form:

$$y_t = I_{\{S_t=1\}}\mu_1 + I_{\{S_t=2\}}\mu_2 + \ldots + I_{\{S_t=K\}}\mu_K + \varepsilon_t \tag{1}$$

where $\varepsilon_t \sim \mathrm{N}(0, \sigma_t^2)$, and

$$\sigma_t^2 = I_{\{S_t=1\}}\sigma_1^2 + I_{\{S_t=2\}}\sigma_2^2 + \ldots + I_{\{S_t=K\}}\sigma_K^2 \tag{2}$$

$$I_{\{S_t=i\}} = \begin{cases} 1 & S_t = i \\ 0 & S_t \neq i. \end{cases}$$

The parameters of the above model are estimated by expectation maximization (EM) algorithm, which is presented in the next subsection. As the result of estimation procedure, among others, one receives two probabilities. The inference probability, which is equal to probability that in time t the Markov chain is in the state i on the condition that the sample up to the observation t is known. In turn the smoothed probability is equal to the probability that in time t the Markov chain is in the state i on the condition that the whole sample is known. Both probabilities are used in the process of state identification.

3.2 Estimation

In this study the parameters of Markov switching models are estimated by EM algorithm. This iterative method originally developed by Dempster et al. (1977) was adjusted by Hamilton (1994) and Kim (1994), for Markov switching models. In order to discuss the application of an EM algorithm to a Markov switching mixture of normal distributions, the following notation has to be introduced. The vector of parameters to estimation is defined as $\boldsymbol{\theta} = (\boldsymbol{\alpha}, \boldsymbol{\lambda})$, where $\boldsymbol{\alpha}$ is defined as $\alpha = \left[\left\{\mu_i, \sigma_i^2\right\}\right]_{i=1,2...K}$. Thus, it contains parameters that determine the normal distribution in each state. In turn, $\boldsymbol{\lambda}$ is a K(K-1)-vector, whose elements correspond to probabilities p_{ij} from matrix **P**. This particular number of parameters is sufficient to describe uniquely transition matrix **P**. To estimate the parameters of vectors $\boldsymbol{\lambda}$ and $\boldsymbol{\alpha}$, one has to calculate inference (filtered), forecast and smoothed probabilities which are defined as $P(S_t = j|\Lambda_t;\boldsymbol{\theta})$, $P(S_t = j|\Lambda_{t-1};\boldsymbol{\theta})$, $P(S_t = j|\Lambda_T;\boldsymbol{\theta})$, respectively, for each state of the Markov chain. is a set of all past observations to the observation t. $\boldsymbol{\theta}^l$ is the whole vector of parameters which is known after l-iterations. By collecting these forecast, inference, and smoothed probabilities in K -vectors, one obtains $\xi_{t|t-1}$, $\xi_{t|t}$, and $\xi_{t|T}$ respectively.
Thus,

$$\begin{aligned}
\xi_{t|t-1} &= (P(S_t = 1 \mid \Lambda_{t-1};\boldsymbol{\theta}^{l-1}), P(S_t = 2 \mid \Lambda_{t-1};\boldsymbol{\theta}^{l-1}), \ldots P(S_t = K \mid \Lambda_{t-1};\boldsymbol{\theta}^{l-1}))' \\
\xi_{t|t} &= (P(S_t = 1 \mid \Lambda_{t};\boldsymbol{\theta}^{l-1}), P(S_t = 2 \mid \Lambda_{t};\boldsymbol{\theta}^{l-1}), \ldots P(S_t = K \mid \Lambda_{t};\boldsymbol{\theta}^{l-1}))' \\
\xi_{t|T} &= (P(S_t = 1 \mid \Lambda_{T};\boldsymbol{\theta}^{l-1}), P(S_t = 2 \mid \Lambda_{T};\boldsymbol{\theta}^{l-1}), \ldots P(S_t = K \mid \Lambda_{T};\boldsymbol{\theta}^{l-1}))'
\end{aligned} \tag{3}$$

The l^{th}-iteration of the EM algorithm starts with parameters obtained from l-1^{th} - iteration. Next one iterates equation (4) and (5) to estimate $\xi_{t|t-1}$ and $\xi_{t|t}$ for t = 1,...T. The initial vector for iterating of equations (4) and (5) is equal to the vector of ergodic probabilities $\boldsymbol{\pi}$.

$$\hat{\xi}_{t|t-1} = \mathbf{P} \cdot \hat{\xi}_{t-1|t-1} \tag{4}$$

$$\hat{\xi}_{t|t} = \frac{\hat{\xi}_{t|t-1} \otimes \mathbf{f}_t}{\mathbf{1}'(\hat{\xi}_{t|t-1} \otimes \mathbf{f}_t)}, \tag{5}$$

where

$$\mathbf{f}_t = (f(y_t \mid S_t = 1, \Lambda_{t-1}; \boldsymbol{\theta}^{l-1}), f(y_t \mid S_t = 2, \Lambda_{t-1}; \boldsymbol{\theta}^{l-1}), \ldots f(y_t \mid S_t = K, \Lambda_{t-1}; \boldsymbol{\theta}^{l-1}))'$$

and

$$f(y_t \mid S_t = j, \Lambda_{t-1}; \theta^{l-1}) = \frac{1}{\sqrt{2\pi\sigma_j^2}} e^{-\frac{(y_t-\mu_j)^2}{2\sigma_j^2}}$$

The **1** represents an K-vector of 1s, and the symbol $\otimes$ denotes element-by-element multiplication. The log likelihood function for the given sample after l-iterations can also be obtained as a by-product of this algorithm in the form:

$$L(\boldsymbol{\theta}) = \sum_{t=1}^{T} \log(\mathbf{1}'(\hat{\boldsymbol{\xi}}_{t|t-1} \otimes \mathbf{f_t})) \tag{6}$$

Next, for estimating smoothed probabilities collected in the vector $\boldsymbol{\xi}_{t|T}$, one uses an algorithm developed by Kim (1994), $t = T$ -1, T - 2,..., 1, namely:

$$\hat{\boldsymbol{\xi}}_{t|T} = \hat{\boldsymbol{\xi}}_{t|t} \otimes (\mathbf{P}' \cdot [\hat{\boldsymbol{\xi}}_{t+1|T} (\div) \hat{\boldsymbol{\xi}}_{t+1|t}]) \tag{7}$$

where sign (÷) denoted element-by-element division. After vector $\hat{\boldsymbol{\xi}}_{t|T}$ is estimated for each observation t, it is possible to estimate elements of vector (probabilities $p_{ij}^{\,l}$ the elements of transition matrix **P**):

$$\hat{p}_{ij} = \frac{\sum_{t=2}^{T} P(S_t = j, S_{t-1} = i \mid \Lambda_T; \hat{\boldsymbol{\theta}}^{l-1})}{\sum_{t=2}^{T} P(S_t = i \mid \Lambda_T; \hat{\boldsymbol{\theta}}^{l-1})} \tag{8}$$

Smoothed probabilities are also needed to estimate the vector α^l, because:

$$\hat{\mu}_j = \frac{\sum_{t=1}^{T} y_t P(S_t = j \mid \Lambda_T; \hat{\boldsymbol{\theta}}^{l-1})}{\sum_{t=1}^{T} P(S_t = j \mid \Lambda_T; \hat{\boldsymbol{\theta}}^{l-1})}, \tag{9}$$

And

$$\hat{\sigma}_j^2 = \sqrt{\frac{\sum_{t=1}^{T} (y_t - \hat{\mu}_j)^2 P(S_t = j \mid \Lambda_T; \hat{\boldsymbol{\theta}}^{l-1})}{\sum_{t=1}^{T} P(S_t = j \mid \Lambda_T; \hat{\boldsymbol{\theta}}^{l-1})}} \tag{10}$$

In this way one arrives at the complete vector $\boldsymbol{\theta}^t$. The process continues until the change in parameters values between iterations is less than the target convergence criterion c = 10^{-8}, namely $\left\|\hat{\boldsymbol{\theta}}^l - \hat{\boldsymbol{\theta}}^{l-1}\right\| \le c$, where $\|\cdot\|$ denotes an Euclidean norm.

Although the EM algorithm monotonically increases the value of likelihood function in each iteration Hamilton (1990), it can not ensure a convergence of parameters to a global maximum for *a priori* starting parameters. It is due to the fact that the EM algorithm is sensitive to the choice of starting values $\boldsymbol{\theta}^0$. For some it can easily get stuck in local maximum or in saddle points Ané and Libidi (2001).

To avoid this problem, 200 starting vectors $\boldsymbol{\theta}^0$ for a two state model and 500 for a three state model are generated randomly and EM algorithm for each of 200 (500) starting vectors is launched. Among the 200 (500) obtained estimators, that which maximized the log likelihood is a maximum likelihood estimator (compare Rydén et al. (1998), Ané and Libidi (2001)).

3.3 Model specification and evaluation

The application of MSMIXN(K) requires determination of the parameter K, which corresponds to the number of state of Markov chain . On the one hand there are arguments for the Markov mixture of only two normal distributions. The examined financial market can be in the calm or crisis regime. It is expected that the crisis regime is characterized by a higher volatility, and smaller mean of expected returns than the calm regime (see Hamilton (1990), Turner et al. (1989)). On the other hand one can expect that by consideration of MSMIXN(3) better fitting to data can be obtained (see Rydén et al. (1998)).

So one would like to test whether MSMIXN(K) or MSMIXN(K+1) has better descriptive validity. The most common approach assumes likelihood ratio test application. Unfortunately, the testing statistic does not have the customary asymptotic χ^2 distribution. For example, when a single normal distribution is tested against Markov switching mixture of two normal distributions, the elements p_{11} and p_{22} of matrix **P** are unidentified under the null hypothesis, which has a form H_0: $\mu_2 = 0$, $\sigma_2^2 = 0$. For a general discussion of this identification problem see Davis (1977).

In the general case of the test MSMIXN(K) against MSMIXN(K+1), the null hypothesis has the form H_0: $\mu_{K+1} = 0$, $\sigma_{K+1}^2 = 0$. In this case 2K parameters are not identified. Inevitability of statistic distribution for this test encouraged researchers to look for alternative methods, which allowed them to carry out the test. In this study three of four well known approaches for model selection are considered. These methods were used or discussed in number of papers (see among others Smith (2002), Schaller and Norden (1997), Hamilton and Lin (1996), Cecchetti et al. (1990), Turner et al. (1989)), and it motivates their application. For a detailed presentation of the fourth approach see Davis (1987).

Garcia (1998) identified critical values as one alternative method for model selection, while Hansen (1996) proposed p-values determination, by means of a simulation as another method respectively. In this study results of Garcia are implemented in case of the test single normal distribution (what corresponds with MSMIXN(1)) against MSMIXN(2). At the same time the Hansen's method (1996), also used by Cecchtti et al. (1990) is applied in case of the test MSMIXN(2) against MSMIXN(3). This method is based on simulation, which is briefly discussed below. The first step of the simulation is to calculate the LR statistic for a given sample, in form: $LR = 2(L_{MSMIXN(3)}-L_{MSMIXN(2)})$, where $L_{MSMIXN(3)}$ and $L_{MSMIXN(2)}$ is the value of the log likelihood function for the parameters which maximise it. Next follows Cecchetti et al. (1990), 1000 time series are generated with the same length as the given sample and with parameters estimated from the MSMIXN(2) model. Instantaneously, for each of this time series a LR_i statistic is computed where i = 1, 2 ... 1000. If k of these statistics exceed the observed LR (computed for the given sample), then the p-value of the test is estimated by the fraction k+1/1000.

In addition to the above approach to model selection two other methods are applied. The idea of the first of them is based on the assumption, that a model exhibits superiority in fitting to data if its choice is invariant on the size of sample. Therefore K can be selected for only a part of the given sample, but the MSMIXN(K) model will be suitable to describing the whole sample. For examining this attribute of the model, a sequential approach may be used. This approach postulates that for the samples consisting of n_0, n_{0+1}...,T observations respectively, likelihood ratio statistics are computed for a single normal distribution against MSMIXN(2), and the MSMIXN(2) model against MSMIXN(3). Thus, one considers two decision problems. One of them is H_0: $\mu_2(n)=0$, $\sigma_2^2(n)=0$, and the second H_0: $\mu_3(n)=0$, $\sigma_3^2(n)=0$, where $\mu_2(n)$, $\mu_3(n)$, and $\sigma_2^2(n)$, $\sigma_3^2(n)$ are parameters estimated for sample of the size n, and n = n_0,...,T. The likelihood ratio (LR) statistic as function of sample size are respectively defined as: $LR_{12}(n) = 2(L_{MSMIXN(2)}(n)-L_{MSMIXN(1)}(n))$, and $LR_{23}(n) = 2(L_{MSMIXN(3)}(n)-L_{MSMIXN(2)}(n))$. $L_{MSMIXN(1)}(n)$, $L_{MSMIXN(2)}(n)$, and $L_{MSMIXN(3)}(n)$ are values of the log likelihood functions for the parameters estimated by an EM algorithm for a sample which contains n-observations. The constancy of the model's choice can be evaluated by plotting graphs of $LR_{12}(n)$ and $LR_{23}(n)$ statistics against the number of observations. If these functions change not dramatically and its values lie above critical value (in case of K =1 against K =2) then one can conclude that the choice of a model is independent of the size of a given sample.

The comparison of moments is the third approach to the model selection problem. By evaluating the difference between the third, and fourth moment estimated from the data (as in Table 1) and the corresponding moments of the Markov switching model, one can make assessment of fitting the model to real market data. The formulae for variance skewness, and excess kurtosis for the MSMIXN(K) model are reported by Timmermann (2000). Below the formulae equivalent to them are presented.

$$Variance = \mu^2 + \sum_{i=1}^{K} \pi_i (\sigma_i^2 + (\mu_i^2 - 2\mu\mu_i)) \tag{11}$$

$$Skewness = \frac{\sum_{i=1}^{K} \pi_i [(\mu_i - \mu)^3 + 3\sigma_i^2 (\mu_i - \mu)]}{[\mu^2 + \sum_{i=1}^{K} \pi_i (\sigma_i^2 + (\mu_i^2 - 2\mu\mu_i))]^{3/2}} \tag{12}$$

$$Excess\ kurtosis = \frac{\sum_{i=1}^{K} \pi_i [(\mu_i - \mu)^4 + 3\sigma_i^4 + 6\sigma_i^2 (\mu_i - \mu)^2]}{[\mu^2 + \sum_{i=1}^{K} \pi_i (\sigma_i^2 + (\mu_i^2 - 2\mu\mu_i))]^2} - 3 \tag{13}$$

where $\boldsymbol{\pi} = (\pi_1, \pi_2, \ldots, \pi_K)$ are ergodic probabilities for the Markov chain S_t which generates states for MSMIXN(K) model, and $\mu = \sum_{i=1}^{K} \pi_i \mu_i$. Despite the complicated form of the above, the equations can be easily derived from the reasoning of Timmermann (2000).

4. Results

According to results reported in Table 1 monthly returns on European indices display higher values of excess kurtosis and skewness than allowed by normal distribution. However, tests for normality do not reject the null hypothesis. The only exception is the main German index. Therefore the analysis starts with the hypothesis that these returns on all indices are normally distributed. Next, one runs the likelihood ratio test to verify descriptive validity of a single normal distribution (MSMIXN(1)) against the MSMIXN(2) model. The values of statistics are reported in Table 2. The null hypothesis is rejected for almost all indices; the only exception is the CAC40, with a LR statistic between the asymptotic and empirical critical values for significant level at 5% reported by Garcia (1998).

Table 2. Results of Tests for Model Selection

Index	Normal v.s MSMIXN(2) Value of statistic	ISMIXN(2) v.s MSMIXN(3) p-value
DAX	23.72***[1]	0.094
CAC40	10.78*[2]	0.037
FTSE100	26.27***	0.007
BUX	15.53***	0.674
PX50	14.42***	0.019
WIG	15.78***	0.742

*[1] *** denotes significance at the level 1 %, the critical value for this level is equal to 14.02.*
*[1] * denotes significance at the level 10%, the critical value for this level is equal to 8.92.*

By applying the EM algorithm, the parameters of the MSMIXN(2) model for each index are obtained. Next for identification of distribution (state) from which each monthly observation has been drawn. Smoothed and inference probabilities are plotted against time for each index. Additionally, on these plots, the duration of international crises are denoted by shaded areas. The following crises are taken into consideration: Asian 06.1997-10.1997, Russian 08.1998-10.1998, Brazilian 01.1999-03.1999, Turkish 02.2001-03.2001, 11th September terrorist attack 09.2001-10.2001, Argentinean 12.2001-02.2002, and Accounting scandals in USA 06.2002-07.2002. From Figure 1 and Table 3 the state 1 for BUX, and state 2 for WIG correspond to unstable market. From figures like Figure 1 and from Table 3, one can conclude that state 1 for DAX, CAC40 and state 2 for FTSE100 and PX50 describe those market in crisis time.

Table 3. Result of Estimations MSMIXN(2) Model for Indices

Parameters	*DAX*	*CAC40*	*FTSE100*	*BUX*	*PX50*	*WIG*
μ_1	-1.023	-1.952	1.522	-4.179	4.072	0.337
	(0.175)	(1.387)	(0.286)	(0.890)	(0.772)	(0.140)
σ_1^2	68.26	56.87	3.724	433.9	2.544	53.77
	(21.74)	(21.01)	(1.875)	(163.6)	(1.811)	(11.95)
p_{11}	0.939	0.773	0.965	0.726	0.624	0.896
	(0.025)	(0.210)	(0.026)	(0.134)	(0.239)	(0.120)
$\mu_{2,}$	2.477	2.122	-0.333	2.197	-0.602	4.973
	(0.549)	(3.240)	(0.587)	(1.076)	(0.975)	(2.385)
σ_2^2	6.135	19.98	21.72	74.71	59.21	404.7
	(2.735)	(5.952)	(3.823)	(19.56)	(9.653)	(214.7)
p_{22}	0.898	0.862	0.983	0.978	0.935	0.000
	(0.043)	(0.120)	(0.014)	(0.031)	(0.051)	(0.210)
Likelihood	-306.67	-301.53	-255.43	-349.51	-317.68	-341.53

Standard errors in parentheses.

One of the striking observations from the results reported in Table 3 is that the probability of staying in a unstable state is higher for indices of western European stock exchanges. It supports the hypothesis that on central European markets trends are less stable than in western Europe. Nonetheless, unstable states for emerging markets exhibit much higher variance in comparison to mature ones. A negative mean is a characteristic for almost all unstable states, however, for the index of the Polish stock market this mean is positive.

The result of moment comparison reveals the superiority of the MSMIXN(2) to a single normal distribution. Actually, for almost all time series, the values of skewness and excess kurtosis for the MSMIXN(2), computed according to Timmermann's formulae, are significantly closer to these parameters estimated directly from the data (see Table 4). Nevertheless, skewness and excess kurtosis for the main French index computed using the above mentioned formulae differ more from the value estimated from the data than in the case of normal distribution. It is a further argument, which supports the finding, that the normal distribution fits better to monthly returns of CAC40 than the MSMIXN(2) model. Therefore, the returns on CAC40 are excluded from further examination, because single normal distribution is selected as best fitting. However, the results of fitting MSMIXN(3) to this index are reported. It is worth emphasising that the variance of MSMIXN(2) model differs only 2.3% on the average from the variance estimated directly from data (see Table 4).

Table 4. Moments for MSMIXN(2)

Moments	*DAX*	*CAC40*	*FTSE100*	*BUX*	*PX50*	*WIG*
Variance	47.88	37.60	16.59	104.7	53.61	88.66
Skewness	-0.459	-0.455	-0.311	-0.630	-0.057	0.727
Excess kurtosis	1.059	0.784	0.670	2.987	0.197	4.411

A further argument supporting the superiority of the MSMIXN(2) is provided by examining the test of LR statistics constancy. The upper graphs in Figure 2 show the $LR_{12}(n)$ for each index for the period from January 1995 to November 2002 along with the asymptotical critical value (5%) reported by Garcia (1998). From upper graphs in Figure 2 it is clear that the MSMIXN(2) exhibit better descriptive properties than a single normal distribution. Almost all the time all plots of $LR_{12}(n)$ lie above the critical value; thus the normality is rejected in favour of MSMIXN(2). There is one more interesting observation to be made from the upper left graph in Figure 2, namely a striking similarity between the graphs of the $LR_{12}(n)$ statistics for western European indices, which could be explained by a strong relation between those markets. For central European indices there are no such similarities in the upper right graph; this is not surprising, since the correlation between these markets is less then half as much as for western ones[3].

In the next part of the analysis, the descriptive validity of the MSMIXN(3) model is assessed. From the right part of Table 2, where p-values are estimated, it is clear that the MSMIXN(2) model is rejected for the FTSE100 and the PX50 at the 5% significance level and for the German main index at 10%. However, for the WIG and BUX, with p-values greater than 0.4, there is no reason to reject the null hypothesis.

3. The correlation coefficients between markets are available upon request from the author.

According to the above findings MSMIXN(3) can be used to model returns on DAX, FTSE100, PX50. However, results of estimation parameters of MSMIXN(3) for the remaining indices are also reported in Table 5. The first observation is easy to make; that consideration of the MSMIXN(3) model instead of the MSMIXN(2) results in a reduction of variance in all states. Nevertheless, the effect of higher variance in unstable states for emerging markets remains. To see this compare state 2 for the DAX and FTSE100 with state 1 for the PX50. It is also interesting to note that the state with the highest absolute mean has relatively low variance. For example state 3 for FTSE100 and state 2 for PX50 have that attribute.

Table 5. Result of estimations MSMIXN(3) Model for Indices

Parameters	*DAX*	*CAC40*	*FTSE100*	*BUX*	*PX50*	*WIG*
μ_1	2.677	2.029	1.208	3.293	-1.022	5.885
	(0.853)	(1.387)	(0.286)	(0.432)	(0.362)	(1.789)
σ_1^2	9.132	22.886	3.225	33.51	64.75	8.350
	(2.119)	(5.038)	(1.875)	(8.532)	(22.73)	(2.368)
μ_2	0.352	-9.879	-0.333	3.302	4.601	-0.977
	(1.559)	(3.240)	(0.587)	(1.076)	(0.975)	(0.167)
σ_2^2	46.29	9.114	21.54	311.5	1.652	59.77
	(16.74)	(5.953)	(5.721)	(83.17)	(0.453)	(16.35)
μ_3	-15.57	-12.99	4.014	-11.99	-0.091	-0.087
	(3.109)	(2.874)	(1.592)	(2.471)	(0.024)	(0.021)
σ_3^2	35.748	3.531	0.631	10.30	0.365	0.593
	(7.391)	(1.295)	(0.736)	(2.633)	(0.053)	(0.152)
Likelihood	-302.23	-293.44	-249.61	-347.49	-308.51	-340.12

Standard errors in parentheses.

For state identification, a procedure analogous to that used for the MSMIXN(2) model is carried out. However, this time, one has to plot two graphs in order to identify the states. From graphs b) and c) in Figure 3 and Table 5 for DAX, FTSE100, and PX50 one knows that states 2 and 3 for the DAX, and state 1 for PX50 and state 2 for FTSE100 correspond to unstable markets.

Table 6 reports the recognizable difference in transition matrices obtained for central and western European indices. The probability of staying in each state for western Europe indices is incomparably higher than for staying in a state for central Europe (see Table 6). This is a further argument for instability trends in central Europe, because also the findings in case of fitting the MSMIXN(2) confirm it.

Table 6. Transition Matrix for MSMIXN(3) Model Estimated for Indices

Index	Transition matrix	Standard errors for main diagonal of transition matrix
DAX	0.885 0.000 0.708	0.0193
	0.039 0.938 0.291	0.0146
	0.076 0.062 0.001	0.1194
CAC40	0.948 0.000 0.955	0.0089
	0.000 0.634 0.025	0.0117
	0.052 0.366 0.020	0.0574
FTSE100	0.882 0.007 0.488	0.0154
	0.000 0.987 0.261	0.0037
	0.118 0.006 0.251	0.0834
BUX	0.727 0.760 0.360	0.0375
	0.195 0.000 0.445	0.0283
	0.078 0.240 0.195	0.0719
PX50	0.814 0.439 0.967	0.0137
	0.156 0.368 0.016	0.0353
	0.030 0.193 0.017	0.0719
WIG	0.000 0.112 0.251	0.0026
	1.000 0.781 0.748	0.0503
	0.000 0.107 0.001	0.3019

In turn the examination of LR constancy does not give an outcome which is easy to interpret. Test of LR constancy statistics did not support the hypothesis that the descriptive validity of the MSMIXN(3) is always better than MSMIXN(2). Because, the $LR_{23}(n)$ is far from being a constant function. Also, there are no strong similarities between lower graphs in Figure 2, as in the case of MSMIXN(2) model.

A comparison of the moments calculated for MSMIXN(3) model with the moments estimated for the given data supports the hypothesis that for any single index, one can not claim *a priori* that the MSMIXN(3) model fits the data better than the MSMIXN(2). In Table 7 moments of the MSMIXN(3) model estimated for each European index are reported. There is a significant improvement in fitting of moments in case of the DAX and PX50. The third and fourth moment is closer to the corresponding one estimated directly from data. Therefore, for these indices the Markov switching mixture of two normal distributions is rejected in favor of the MSMIXN(3) model. In turn, in the case of the main Hungarian index the comparison leads to a selection MSMIXN(2). It should be noted that comparison of moments in the case of MSMIXN(3) also can lead to the situation in which it is difficult to say if considering this particular model results in substantial improvement in data fitting.

For example, there is an improvement in fitting the excess kurtosis and worsening in case of the skewness for FTSE100, and vice versa for WIG.

Table 7. Moments for MSMIXN(3)

Moments	*DAX*	*CAC40*	*FTSE100*	*BUX*	*PX50*	*WIG*
Variance	49.27	33.28	18.02	108.4	53.87	90.44
Skewness	-0.926	-0.562	-0.244	0.109	-0.359	0.450
Excess kurtosis	1.682	0.582	0.394	2.487	0.434	5.051

Finally, it is worthwhile to summarize the results. A single normal distribution has the best descriptive validity for the CAC40. The MSMIXN(2) model fits well for the BUX and WIG, and the MSMIXN(3) exhibits superiority in describing returns of the DAX, FTSE100, and PX50.

In order to measure the leverage effect on the mature and emerging markets in Europe, the parameters of the returns distribution during the crisis with parameters of distribution during the calm period are compared. The results indicate that in the case of MSMIXN(2), the average difference between the standard deviation describing the market in crisis periods and the standard deviation describing the market in calm periods for western European markets is equal to 3.84. However, for central Europe this magnitude amounts to 10.4. A similar result is obtained in the case of MSMIXN(3), namely the average difference is equal to 3.51 for the mature and 9.61 for the emerging markets. Consequently, one observes that when the European markets being examined exercise a downturn, the returns on them are drawn from a normal distribution characterized by higher variance. It confirms the presence of the leverage effect on the examined markets. In order to examine the importance of the leverage effect for emerging and mature markets, the above values are divided by corresponding average standard deviation of returns observed for each group of the markets. In case of both models MSMIXN(2) and MSMIXN(3) the results indicate that during the crisis the leverage effect is more visible on the emerging than on the mature markets.

For evaluating the vulnerability of markets to international crises, once more Figures 1 and 3 are examined. The CAC40 is excluded from the analysis, because its returns are drawn from a single normal distribution. It should also be stressed that by considering monthly returns the analysis concentrates on crises, which have a substantial impact on the economy of the examined country. According to graphs b) and c) in those Figures the Russia crisis had the most important influence on all European markets, especially on markets in central Europe. In the first phase of this crisis the returns drop to -17.5%, -11.5%, -10%, -43%, -24%, and -31% for the DAX, CAC40, FTSE100, BUX, PX50, and WIG, respectively. In case of Asian crises, the most significant decrease in returns took place on the Frankfurt and London stock exchanges. However, the influence of this crisis on emerging European markets was relatively limited. In contrast to western European markets, the September 11th terrorist attack did not have significant influence on monthly returns on indices from central Europe. The accounting scandal had impact on the DAX, FTSE100, and PX50 but its influence on monthly returns of the BUX and WIG was negligible. The Markov mixture of normal distributions did not detect any significant impact of the other international crises considered on monthly returns from emerging markets.

5. Conclusions

This paper demonstrates that Markov switching mixture of two or three normal distributions, namely MSMIXN(2) and MSMIXN(3) can successfully describe monthly returns on indices from western and central Europe. Both of these models are able to capture important events of international financial crises in the period from January 1995 to November 2002. The results also indicate that these models are able to reproduce the properties of monthly returns, namely the values of the excess kurtosis and skewness.

Additionally the test of likelihood ratio statistic constancy, as well as customary version of likelihood ratio test reveals the superiority of the MSMIXN(2) to a single normal distribution in modelling returns. However in the case of choice between MSMIXN(2) and MSMIXN(3) the results indicate that one cannot *a priori* decide if the MSMIXN(3) model fits better to a given sample than the MSMIXN(2).

The investigation shows that the leverage effect is observed on both mature and emerging markets in Europe. Nonetheless, the change from the calm to the crisis regime leads to the larger increase in volatility on the emerging markets. The further finding is that the market trends on stock exchanges in central Europe are less stable than in western Europe. Finally, evaluating the vulnerability of emerging markets from this region to international crises shows that the Russian crisis had a substantial impact on them.

Figure 1. The graphs a) present monthly return on BUX and WIG index; the graphs b) contain the filtered (dashed line) and smoothed (solid line) probability.

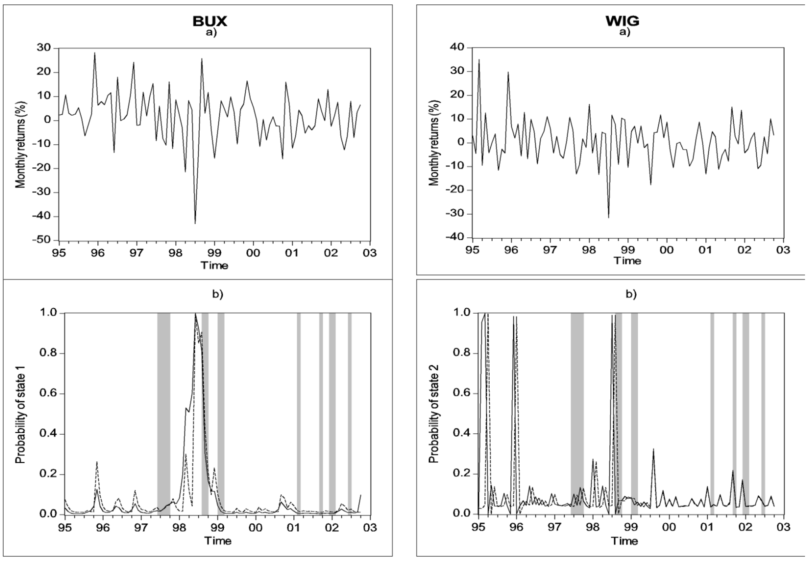


Figure 2. The upper graphs present statistic $LR_{12}(n)$ and the lower graphs present statistic $LR_{23}(n)$ for DAX, FTSE100, CAC40 and BUX, PX50, WIG respectively.

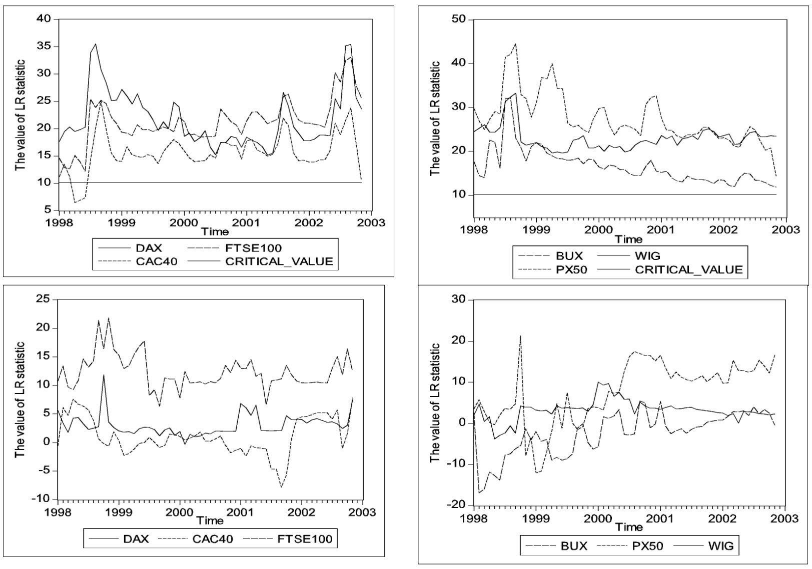
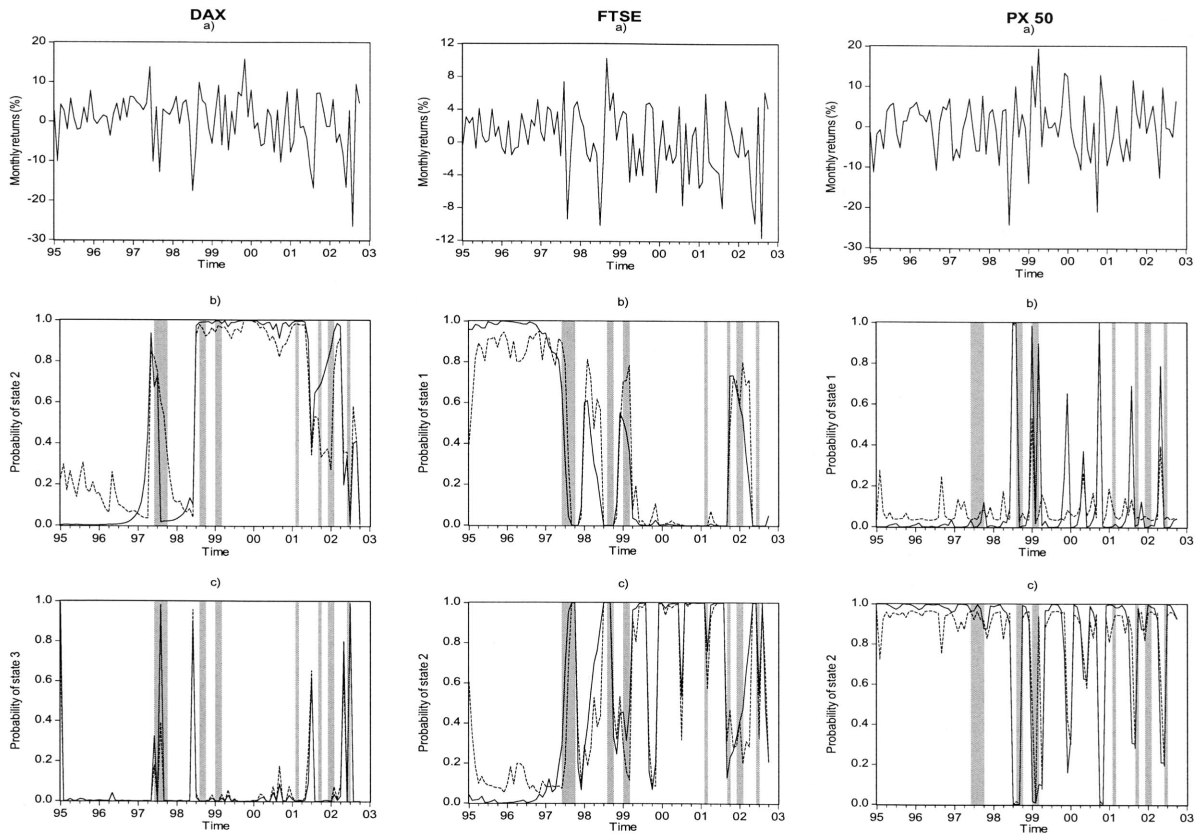


Figure 3. The graphs a) present monthly returns on DAX, FTSE100, and PX50 index; respectively the graphs b) and c) contain the filtered (dashed line) and smoothed (solid line) probability.



References

Ané, Th., Labidi Ch., 2001, *"Revisiting the Finite Mixture of Gaussian Distributions with Application to Future Markets"*, Journal of Futures Markets 21, 348-376.

Ang, A., Bekaert, G., 2002, *"International Asset Allocation with Regime Shifts"*, Journal of Empirical Finance 15, 1137-1187.

Bekaert, G., Harvey, C.R., 2003, *"Emerging Markets Finance"*, Journal of Empirical Finance 10, 3-55.

Black, F., 1976, *"Studies of Stock Market Volatility Changes"*, Proceedings of the 1976 Business Meeting of the Business and Economic Statistics Section, American Economic Association, 177-181.

Blattberg, R., Gonedes N., 1965, *"A Comparison of the Stable and Student distributions as Statistical Models for Stock Prices"*, Journal of Business 47, 244-280.

Cecchetti, S., Lam, P., Mark, N., 1990, *"Mean Reversion in Equilibrium Asset Prices"*, The American Economic Review 80, 398-418.

Chunhachinda, P., Dandapan, K., Hamid, S., Prakash, A.J., 1997, *"Portfolio Selection and Skweness: Evidence from International Stock Markets"* Journal of Banking and Finance 21, 143-167.

Davies, R.B., 1987, *"Hypothesis Testing when the Nuisance Parameter is Present only under the Alternative"*, Biometrika 74, 33-43.

Davies, R.B., 1977, *"Hypothesis Testing when the Nuisance Parameter is Present only under the Alternative",* Biometrika 64, 247-25.

Engle, R.F., Ng, V.K., 1993, *"Measuring and Testing the Impact of News on Volatility"*, Journal of Finance 48, 1749-1778.

Dempster, A.P., Laird, N.M., Rubin, D.B., 1977, *"Maximum Likelihood from Incomplete Data via EM Algorithm"*, Journal of the Royal Statistical Society 39, 1-38.

Fama, E., 1965, *"The Behaviour of Stock-market Prices"*, Journal of Business 38, 34-105.

Garcia, R., 1998, *"Asymptotic Null Distribution of the Likelihood Ratio Test in Markov Switching Models"*, International Economic Review 39, 763-788.

Hamilton, J., Lin, G., 1996, "Stock Market Volatility and the Business Cycle", Journal of Econometrics 11, 573-593.

Hamilton, J., 1994, "*Time series"* , Princeton University Press Princeton, New Jersey, 677-703.

Hamilton, J., 1990, *"Analysis of Time Series Subject to Changes in Regime"*, Journal of Econometrics 45, 39-70.

Hansen, B.E., 1996, *"The Likelihood Ratio Test under Non-standard Assumptions: Testing the Markov Switching model GNP"*, Journal of Applied Econometrics 7, S61-S82; erratum 11, 195-8.

Harvey, C.R., 1991, *"The World Price of Covariance Risk"*, Journal of Finance 46, 111-157.

Harvey, C.R., 1995, *"Predictable Risk and Returns in Emerging Markets"*, Review of Financial Studies 8, 773-815.

Kim, Ch., 1994, *"Dynamics Linear Models with Markov-switching"*, Journal of Econometrics 60, 409-432.

Kitazawa, Y., 2000, *"Estimating the Leverage Effect using Panel Data with a Large Number of Stock Issues over a Short-Run Daily Period Focus on the Tokyo Stock Exchange"*, Journal of Financial Management & Analysis 13, 21-28.

Kon, S., 1984, *"Models of Stock Returns-comparison"* , Journal of Finance 36, 149-165.

Koutmos, G., Saidi R., 1995, *"The Leverage Effect in Individual Stocks and the Debt to Equity Ratio"* , Journal of Business Finance & Accounting 22, 1063-1076.

Köke, J., 1999, *"Institutional Investment in Central and Eastern Europe: Investment Criteria of Western Portfolio Managers"*, ZEW discussion paper No. 99-37.

Lai, T.Y., 1991, *"Portfolio Selection with Skewness: a Multiple-objective Approach"*, Review Quantitative Finance Accounting 1, 293-305.

Linne, Th., 2002, *"A Markov Switching Model of Stock Returns: an Application to the Emerging Markets in Central and Eastern Europe"*, East European Transition and EU Enlargement, Physica-Verlag Heidelberg New York, 371-384.

Peiró, A., 2002, *"Skewness in Individual Stocks at Different Investment Horizons"*, Quantitative Finance 2, 139-146.

Rydén, T., Teräsvirta, T., Ásbrink, S., 1998, *"Stylised Facts of Daily Returns Series and the Hidden Markov Model"*, Journal of Applied Econometrics 13, 217-243.

Schaller H., Norden S., 1997, "Regime Switching in Stock Market Returns", Applied Financial Economics 7, 177-192.

Schwert, W.G., 1990, *"Stock Volatility and the Crash of 87"*, The Review of Financial Studies 3, 77-102.

Smith D.R., 2002,*"Markov-Switching and Stochastic Volatility Diffusion Models of Short-Term Interest Rates"*, Journal of Business and Economic Statistics 20, 183-197.

Timmermann, A., 2000, *"Moments of Markov Switching Models"*, Journal of Econometrics 96, 75-111.

Turner, Ch., Stratz, R., Nelson, Ch., 1989, *"A Markov Model of Heteroskedasticity, Risk, and Learning in the Stock Market"*, Journal of Financial Economics 25, 3-22.