ASECU South-Eastern Europe Journal of Economics 2 (2010) 187-226 SEEJE

INTERNATIONAL POSITIVE PRODUCTION EXTERNALITIES AND CAPITAL MOVEMENT

CHARALAMPOS SAVVIDIS
Panteion University

Abstract
I attempt to establish whether positive international externalities generate an incentive for cooperation between governments and how that incentive depends on the degree of capital mobility between economies. I adopt a simple economic model incorporating the international linkage of national economies.

Allowing for capital mobility does not destroy the incentive to cooperate, since it does not affect optimal policies in the symmetric long run equilibrium. In the short run, capital mobility triggers a 'race to the bottom' effect, which comes in addition to the 'free riding' effect existing because of positive international production spillovers. Thus, allowing for capital mobility intensifies the need to cooperate.

JEL Classification: F21; F42; H23.

Key words: Production Externalities, Capital Mobility, International Policy Coordination, Intertemporal Choice.

Corresponding address: Charalampos Savvidis, 2, Keramiku str., Athens 15125
e-mail: hsav@hotmail.com

1. Introduction

A prominent feature of the present phase of the globalisation process is the increased interconnection between economies: Domestic production decisions affect production (hence, decisions as well) abroad. One may identify several episodes of positive production spillovers between countries: building roads or ports in one country may help companies in other countries to export their products. Developing a modern telecommunication network (e.g. providing ADSL network access) may be of equal importance.

The existence of international externalities should generate an incentive for international cooperation between different governments: according to the literature, in the presence of positive spillovers, players' actions increase when switching from an uncoordinated to a coordinated equilibrium[1]. As long as governments stay on the rising part of the Laffer curve, such an increase implies a welfare improvement, providing governments an incentive to cooperate.

A second feature of modern economy is the record-high degree of capital mobility[2]. Investors allocate their capital on a global level (through either foreign direct investments or portfolio investments) so as to attain the highest feasible returns. For capital to be attracted, economies should offer high potential returns. Thus, governments adopt pro-capital policies and tend to refrain from any decisions investors would dislike. Increasing capital taxation is an obvious disincentive to investors. Governments are, thus, not able to take fiscal decisions without considering implications on capital mobility. Moreover, governments try to attract capital by decreasing tax rates – a stand that may end in a 'race to the bottom' competition.

In the present paper, I try to answer how the existence and degree of tax competition influences the incentive to cooperate in a framework with international spillovers from public services. The next section (section 2) summarizes related literature and describes the present paper's additions to it.

Section 3 describes the environment adopted in the paper. A technical approach within a pure neoclassical growth framework is assumed. A public good, financed by tax revenues, affects production positively both at home and abroad. I will also assume some degree, θ, of capital mobility. The Competitive Decentralized Equilibrium will also be described. In order to incorporate the international linkage of national

1. For example, see the seminal paper of Cooper and John (1988).

2. According to UNCTAD, in 1990 FDI flows at world level were \$201,6 billion and the global stock was \$1779,2 billion. In 2006 (before the onset of the financial crisis) FDI flows increased to \$1305,8 billion while their global stock reached \$11998,8 billion. For more details see: http://stats.unctad.org/FDI/TableViewer/tableView.aspx?ReportId=899.

economies, I adopt a form presented by Alesina & Wacziarg (1999)[3], considering it as more applicable to a standard Cobb-Douglas function.

I then (section 4) solve governments' optimisation problem when they play Nash to each other and when they cooperate, adopting a common tax rate (tax harmonization). In both cases, I find the long run equilibrium values and study dynamics around it. In the 5th section I compare the results of both cases and try to understand how the difference between the two is affected by the degree of capital mobility. Finally, section 6 summarizes the main results and suggests ideas for future research.

2. Relationship to the literature

The literature has concluded that international externalities generate an incentive for international cooperation between governments. Philippopoulos and Economides (2003) develop an endogenous growth model with public goods, comparing optimal taxes in the Nash equilibrium and in the case of a cooperative solution. Epifani and Gancia (2008) recently studied the effect of trade openness on the size of governments. Their claim is that "Since governments behaving non-cooperatively do not internalize the cost of taxation that trade imposes onto foreigners, they react to market integration by increasing public spending". The authors demonstrate that "when all governments raise spending in response to more trade openness, the result is overprovision of public goods". In this context, international coordination of tax policies may be welfare improving.

Recent literature has also explored the question whether, in the case of public input provision, tax competition is good for growth. Following the seminal work of Oates (1972), Zodrow and Mieszkowski (1986) demonstrated formally that public services are inefficiently provided under tax competition. Local governments keep tax rates low, hoping to expand the tax base. Noiset (1995) disagreed with them, noting that while a higher tax rate itself drives out capital, the tax base may be expanded if the revenue raised is spent on increasing capital's productivity. Local governments competing for capital may, thus, have an incentive to overprovide public inputs – a possibility not considered in Zodrow and Mieszkowski. Matsumoto (1998) showed, however, that potential overprovision is of limited importance, since it becomes invalid when relaxing Zodrow and Mieszkowski model's silent assumption, that the number of firms in each jurisdiction is fixed.

Becker and Rauscher (2007) present an interesting approach, building on the seminal work of Lejour and Verbon (1997). They assume capital installation costs, mak-

3. Alesina and Wacziarg (1999) present a model with cross-country externalities, where output depends on labor, private capital and the weighted product of public services across the world. Governments impose a proportional tax on output to finance the provision of the public service.

ing de-installation of capital more costly and thus allowing governments to exploit an increasingly inelastic tax base. In such a framework, "an individual government neglects the impact of its tax policy on the interest rate. If all countries do this, the asset market equilibrium collapses"[4]. Policy cooperation may, thus, arise as a solution. The authors suggest in their final remarks that "future research could aim at comparing tax competition to a coordinated tax policy".[5]

Koethenbuerger and Lockwood (2007) study the relationship between tax competition and growth in an endogenous growth model, where there are stochastic shocks to productivity, and capital taxes fund a public good, which may be for final consumption or an infrastructure input. They predict a negative relationship between output volatility and growth, which they find consistent with the empirical evidence.

Philippopoulos and Kammas (2007) reexamine the quantitative welfare implication of international tax cooperation, incorporating an international public good in a multi-country version of the general equilibrium model in Persson and Tabellini (1992). Two types of cross-border spillovers arise: One generated by international capital mobility and resulting in the problem of tax competition for mobile tax bases. And a second, generated by the presence of international public goods and resulting in the problem of free riding on other countries' contribution. As they demonstrate, while in the absence of international public goods, the welfare gain from cooperation is small quantitatively, results change drastically once they introduce international public goods. Their main conclusion is that the argument for international cooperation becomes much stronger when there are public goods that extend beyond national borders.

Hatfield (2006) illuminates how tax competition drives the districts of a federal economy to choose better economic policies in their quest to acquire and keep capital, concluding that federalism leads to higher economic growth, in a framework of endogenous growth with government services. Bjorvatn and G. Schjelderup (2000) demonstrate that allowing for capital mobility in the presence of a locally provided international public good, may not affect the need to cooperate in a symmetric equilibrium.

Assuming a public infrastructure good (rather than a public consumption good) and using a dynamic model, I will try to answer whether capital mobility works as a substitute to cooperation or if it intensifies the need for it, i.e. how the existence and degree of tax competition influences the incentive to cooperate in a framework with international spillovers from public services. I will establish a simple economic model incorporating the international linkage of national economies, in order to

4. Becker and Rauscher (2007, p. 14).

5. Becker and Rauscher (2007, p. 18).

check whether an incentive for cooperation in setting national policies exists. Imperfect capital mobility will also be assumed, in order to test its implications on fiscal decisions. The present work is, hence, closer to the framework developed by Philippopoulos and Park (2002).

I differentiate from existing literature assuming a public infrastructure good rather than a public consumption good. In the specific adopted framework, a 'free riding' effect arises, creating an incentive for governments to cooperate in choosing their tax policies. Allowing for capital mobility, this gives birth to a 'race to the bottom' effect and a 'tax the foreigner effect'. The paper adds to the literature by putting together all those effects under a dynamic framework and checking how the incentive for cooperation in setting national policies is affected.

3. The environment

3.1 A framework with international public services

Consider a continuous time model, where a given finite number of countries, n, indexed by i=1,2,...,n, exist. Production depends on a public production factor, G, that is non-rival and non-excludable for all n economies – an international public service. The extent of the externality goes beyond the frontiers of each country, namely, there are cross-country externalities, so that each country benefits from public goods produced in the rest of the world. Those externalities may be understood as public infrastructures, benefiting the international linkage of the economies, such as transport networks, harbours, airports etc.

In order to incorporate the international linkage of national economies, a form presented by Alesina & Wacziarg (1999) is adopted.[6] Alternatively one may use an additive function for the public good. This, however, results in symmetry to per capita income depending directly on the number of economies sharing the public good,[7] while this is not the case when using the product of national tax revenues.[8] Obviously, by using logarithms one form may be transformed to the other. Alesina & Wacziarg (1999) let:

$$Y_j = AL_j^{1-\alpha}K_j^{\alpha}\prod_{i=1}^{n}\left[\left(\omega_i G_i\right)^{\frac{1-\alpha}{n}}\right]$$

6. Alesina and Wacziarg (1999) present a model with cross-country externalities, where output depends on labor, private capital and the weighted product of public services across the world. Governments impose a proportional tax on output to finance the provision of the public service. This formulation is adopted as more applicable to a standard Cobb-Douglas function.

7. In symmetry it turns out to be: $G=(n\tau A)^{1/\alpha}k$, which results in: $y=[(n\tau)^{1-\alpha}A]^{1/\alpha}k$.

8. In the latter case, the Nash tax rate proves to depend on the number of economies sharing the externality. This, however, may be explained as a typical result of a free riding problem.

In the present work a more general form will be adopted, where weights ω_i are used as exponents – an assumption that is also in compliance with the needs of logarithmic transformation.

Production function in country j will, hence, be of the form: $Y_j=A_jK_j^{\alpha}L_j^{1-\alpha}G_j^{1-\alpha}$, where $0<\alpha<1$, Y_j the only good produced, A_j the level of technology in country j, K_j and L_j the employed amounts of capital and labor, respectively and G_j the amount of the international public service affecting economy j.[9] There is no trade between countries and private agents cannot work abroad. There is however one more international linkage, as individuals are allowed to invest either at home or abroad,[10] with the latter decision bearing some extra cost.[11]

3.2 The problem of the representative agent

Each country is populated by 'immortal' (i.e. with infinite planning horizon) identical private agents, who get utility by consuming a single good. For individual i it will be:

$$U_i = \ln(C_i) \tag{1}$$

Since only one good exists, there are no prices. All markets work perfectly competitively. Private agents in each economy possess amounts of capital, K, and labour, L, through which the single good is produced, according to the function:

$$y_i = A_j k_j^{\alpha} G_j^{1-\alpha} \tag{2}$$

For reasons of simplicity labour force is normalized to unity at the economy's level and assumed to be constant over time (i.e. each agent owns one unit of labour at any point of time);[12] thus per capita values (denoted by small letters) will equal absolute values.

A representative private agent, h, in country j wishes to maximize utility intertemporally, choosing its consumption level under the restriction of its income and allocating its assets either at home, $B_{hj,j}$ or abroad $B_{hj,f}$. Net return for agent h of

---

9. This formulation assumes that the flow of government purchases, G, enters into the production function. Including a stock of accumulated public capital may be more realistic. For reasons of simplicity, however, the focus will be on 'flow' government purchases.

10. Note that physical capital, in fact, remains immobile: Once households decide to invest in an economy, physical capital may not be moved. Private assets, however, may be allocated abroad.

11. In the real world perfect capital mobility is usually not the case. There are several 'barriers' to capital mobility, including imperfect information, administrative fees, differing regulations, inability to directly control capital invested abroad etc. Note, however, that the costs of foreign investment may themselves be regarded as policy instrument, as is the case in the literature on capital controls. For an analysis in a political economy context, see Alesina and Tabellini (1989).

12. This is possible due to constant returns to scale. See also Barro & Sala-i-Martin (1995), ch. 2.

country j for his foreign investment will be: $R_{hj,f} = B_{hj,f}(r_f - \frac{\theta}{2} b_{hj,f})$, where θ denotes 'mobility costs'[13] and r_f the interest rate in the foreign economy. The private agent's budget constraint at any point of time will be: $w_j L_{hj} + r_j B_{hj,j} + R_{hj,f} = \dot{B}_{hj,j} + \dot{B}_{hj,f} + C_{hj}$, where r_j denotes the interest rate in economy j, C_j the chosen consumption level, w_j wages in economy j and a dot over a variable symbolizes time derivative. Let B_{hj} denote total assets belonging to household h of economy j, $B_{hj} = B_{hj,j} + B_{hj,f}$.

3.3 The problem of government

In each country there is also a benevolent government, that taxes domestic production to finance the provision of the public production factor. At any point of time, the level of the public production factor depends on public spending in all n economies:

$$G_j = \prod_{i=1}^{n} (g_i^{\omega_{i,j}}) = \prod_{i=1}^{n} (\tau_i y_i)^{\omega_{i,j}} \tag{3}$$

where $0 \leq \tau_j \leq 1$ is the tax rate imposed by government in economy j on domestic per capita income, $\sum_{i=1}^{n} \omega_{i,j} = 1,\ 0 < \omega_{i,j} < 1 \forall i$ and $\omega_{i,j} \leq \omega_{j,j}\ \forall\ i \neq j$. The weight $\omega_{i,j}$ represents the extent to which public spending in country i affects the level of the public factor in country j. Trying to focus on capital mobility's policy, I will focus on the case of two countries (j and f). I assume governments adopt the source taxation principal, i.e. government taxes domestic and foreign investors at the same rate. The sequence of moves is as follows: first national governments choose their tax policy; then private agents make their consumption-investment choices.[14] Policy responsibilities and the sequence of moves are taken as given.

3.4 The World Competitive Decentralized Equilibrium

Household's maximization problem will be:

$$\max_{c_{hj},\, b_{hj,f}} \left\{ \int_0^{\infty} \left[\ln\left(c_{hj}\right) e^{-\rho t} \right] dt \right\},\ \text{s.t. } w_j + r_j b_{hj,j} + b_{hj,f}\left(r_f - \frac{\theta}{2} b_{hj,f} \right) = \dot{b}_{hj} + c_{hj},$$

where the parameter $\rho > 0$ is the rate of time preference and prices, public goods and tax policy are taken as given. At economy level maximization yields to:

---

13. For illustrative tractability $\frac{\theta}{2}$ instead of θ will be used.

14. Because of the timing, there is no credibility problem vis-a-vis the private sector in the choice of the capital tax rate. Persson and Tabellini (1999) provide an extensive discussion of these credibility problems and of how the timing assumed here could be enforced through the design of political institutions that delegate policymaking to an elected official.

$$\frac{\dot{c}_j}{c_j} = r_j - \rho \tag{4}$$

There will also be an arbitrage condition, asking for:

$$b_{j,i} = \frac{r_f - r_j}{\theta} \tag{5}$$

There is also a transversality condition, which states that the value of assets in terms of current utility should approach zero as time approaches infinity:

$$\lim_{t \to \infty} \left(\lambda_j b_j e^{-\rho t} \right) = 0 \tag{6}$$

In Appendix A it is proved that the budget constraint at the economy level will be:

$$\dot{b}_j = w_j + r_j b_j + \frac{\left(r_f - r_j\right)^2}{2\theta} - c_j \tag{7}$$

A representative firm h in country j tries to maximize profits choosing the ratio of capital to labour employed[15] and taking prices, public goods and tax policy as given. The maximization problem will, hence, be:

$$\max_{k_{hj}} \left\{ L_{hj} \left[\left(1 - \tau_j\right) A_j k_{hj}^{\alpha} G_j^{1-\alpha} - w_j - \left(r_j + \delta\right) k_{hj} \right] \right\},$$

where the parameter $\delta \leftarrow 0$ is the rate of capital depreciation. At country level profit maximization yields to:

$$r_j = \alpha \left(1 - \tau_j\right) A_j G_j^{1-\alpha} k_j^{\alpha-1} - \delta \tag{8}$$

Given constant returns to scale there will be no profits at any point of time:

$$w_j = (1 - \alpha)\left(1 - \tau_j\right) A_j k_j^{\alpha} G_j^{1-\alpha} \tag{9}$$

In each economy, government should set the tax rate in order to finance the public production factor, satisfying both economies' budget constraint:

$$G_j = \left(\tau_j y_j\right)^{\omega_j} \left(\tau_f y_f\right)^{1-\omega_j} \tag{3a}$$

$$G_f = \left(\tau_f y_f\right)^{\omega_f} \left(\tau_j y_j\right)^{1-\omega_f} \tag{3b}$$

15. Remember that labour force is normalized to unity at economy's level.

A World Competitive Decentralized Equilibrium16 (WCDE) can now be characterized. This is for any feasible national fiscal policies as summarized by the national tax rates, τ_i and τ_f.

Definition 1

In the World Competitive Decentralized Equilibrium and for any feasible national tax rate: (i) all private agents maximize utility; (ii) all constraints are satisfied; (iii) all markets clear.

In country j there are six unknown variables (G_j, b_j, k_j, c_j, r_j and w_j), which will be determined by equations (3a), (4), (5), (7), (8) and (9). Equations (3a), (5) and (7), however, include also r_f, k_f and G_f, which will be determined by the corresponding system of equations in country f. Finally, there is also the global constraint, demanding capital stock in all economies to equal the sum of all economies' assets:

$$k_j+k_f=b_j+b_f \tag{10}$$

Thus, the problem entails 6*2 equations (i.e. equations (3), (4), (7), (8) and (9) for both countries, the arbitrage condition (5) and the global constraint (10) that will determine values for 6*2 variables. In the symmetric case these equations give closed-form analytical solutions for equilibrium allocations as functions of national tax rates, τ_j.

4. Determination of national tax policies

In this section national policies will be endogenized. Initially, national tax rates, τj and τf, will be determined by a Nash game among benevolent national governments, which try to intertemporally maximize utility in their country. In choosing τj national government in country j takes into account economy j's constraints in a WCDE (specified above). Then I will solve for the case governments cooperate to choose a common tax rate, τ, in order to intertemporally maximize global utility.

4.1 Non-cooperative (Nash) national policies

Government in country j chooses τj to maximize utility subject to its own constraints in a WCDE. In doing so, it takes into account the behaviour of domestic agents, it takes as given τ_f, c_f, k_f and their shadow prices and plays Stackelberg vis-à-vis private agents. Government in country j tries to:

$$\max_{\tau_j} \int_0^\infty \left[\ln\left(c_j\right)e^{-\rho t}\right]dt \quad \text{s.t. equations (2) - (9) above}$$

16. The term 'competitive' suggest that prices are taken as given, while the term 'decentralized' that private agents may not internalize externalities. For a more detailed presentation see Blanchard and Fischer (1989, p. 76).

Without loss of generality, the focus will be on Symmetric Nash Equilibria (SNE) in national policy rules[17]. Symmetry implies countries may differ ex-ante, but after determining policy strategies they become identical, i.e. they will be symmetric ex-post. Thus, in equilibrium, $\tau_j=\tau_f=\tau$, and hence $c_j=c_f=c$, $k_j=k_f=k$. The extent of the contribution of domestic infrastructure will be common for both countries, i.e. $\omega_{j,j}=\omega_{f,f}=\omega$. The extent of the external contribution of domestic infrastructure in foreign production functions will also be common, i.e. $\omega_{f,j}=\omega_{f,j}=1-\omega$. An obvious realistic assumption would be that $\omega>1-\omega \Leftrightarrow \omega>0.5$, i.e. domestic infrastructure affects the economy more strongly than foreign infrastructure does. Invoking symmetry into the first order conditions resulting from government's maximization problem yields Proposition 1, below.

Proposition 1

A Symmetric Nash Equilibrium (SNE) in national tax policies and the associated World Competitive Decentralized Equilibrium is summarized by equations (11a) – (11f) below. Equation (11a) determines the Nash tax rate, denoted as $0<\tau^{\$}<1$, which is unique.

$$\frac{\theta}{2}\left[1+\frac{1-\alpha}{\alpha}(1-\omega)\right](vk+\alpha\mu c)\left[(1-\alpha)\frac{\alpha\omega+(1-\alpha)(1-\omega)}{\alpha+(1-\alpha)(1-\omega)}-\tau\right]+(1-\tau)(1-\alpha)\frac{1}{k}\frac{y}{k}$$

$$\left\{2(1-\omega)(1-\alpha-\tau)(vk+\alpha\mu c)+\alpha vk\left[(1-\alpha)(2\omega-1)-\tau\right]\right\}=0 \tag{11a}$$

$$\dot{k}=\dot{b}=w+rk-c \tag{11b}$$

$$\frac{\dot{c}}{c}=\alpha(1-\tau)\frac{y}{k}-\delta-\rho \tag{11c}$$

$$\frac{\dot{v}}{v}=\rho-r\frac{y}{k}(1-\alpha)(1-\tau)\left\{\frac{\theta}{2}\left[\alpha\omega+(1-\alpha)(1-\omega)\right]+2\alpha(1-\alpha)(1-\tau)(1-\omega)\frac{y}{k}\frac{1}{k}-\alpha\frac{\mu}{v}\right.$$

$$\left.\frac{c}{k}(1-\omega)\frac{\theta}{2}\right\}\left\{\frac{\theta}{2}\left[1-(1-\alpha)(2\omega-1)\right]+4\alpha(1-\alpha)(1-\tau)(1-\omega)\frac{y}{k}\frac{1}{k}\right\}^{-1} \tag{11d}$$

$$\frac{\dot{\mu}}{\mu}=\frac{v}{\mu}-\frac{1}{c}\frac{1}{\mu}-\left[\alpha(1-\tau)\frac{y}{k}-\delta-2\rho\right] \tag{11e}$$

$$\lim_{t\to\infty}\left(e^{-\rho t}vk\right)=0 \tag{11f}$$

Proof: See Appendix B

17. This is a standard practice in the literature. For example see Kehoe (1987, p. 361)

Variables are transformed in order to reduce the dynamic dimensionality of the model and facilitate analytical tractability. Let: $\varphi \equiv \mu k$, $z \equiv \frac{c}{k}$ and $\psi \equiv \frac{\mu}{v}$. In Appendix C it is shown that system (11a)-(11f) is equivalent to the following:

$$\frac{\dot{\tau}}{\tau} = \frac{1-\tau}{\tau}\{[z + \frac{1}{\psi} - \frac{1}{z}\frac{1}{\varphi} - \frac{1}{\alpha}\frac{1}{\tau-(2\omega-1)(1-\alpha)}(1-\alpha)(1-\tau)(1-\omega)(1+\alpha\psi z)$$

$$(1-\alpha-\tau)(A\tau^{1-\alpha})^{1/\alpha}]\frac{1}{\xi}[s_2(\tau^{@}-\tau)+\xi s_3(\widetilde{\tau}-\tau)][\xi s_1(s_4-\tau)-s_2(\tau-\tau^{@})]+[(1-\tau)$$

$$(A\tau^{1-\alpha})^{1/\alpha}-\delta-z]s_2(\tau^{@}-\tau)[s_1(s_4-\tau)+s_3(\tau-\widetilde{\tau})]\}\{s_2\{[s_3(\tau-\widetilde{\tau})+s_1(s_4-\tau)]$$

$$\left[1-\tau^{@}-(\tau-\tau^{@})\frac{1-\alpha}{\alpha}\frac{1-\tau}{\tau}\right]-(1-\tau)(\tau-\tau^{@})(s_3-s_1)\}+s_1 s_3\xi(1-\tau)(s_4-\widetilde{\tau})\}^{-1} \tag{12a}$$

$$\frac{\dot{z}}{z} = z-(1-\alpha)(1-\tau)A^{1/\alpha}\tau^{1-\alpha/\alpha}-\rho \tag{12b}$$

$$\frac{\dot{\psi}}{\psi} = \frac{1}{\psi}+\rho-\frac{1}{z}\frac{1}{\varphi}+(1-\alpha)(1-\tau)A^{1/\alpha}\tau^{1-\alpha/\alpha}\frac{1}{\alpha}\{\tau[\alpha+(1-\omega)(1+\alpha\psi z)]-(1-\alpha)$$

$$[\alpha(2\omega-1)+(1-\omega)(1+\alpha\psi z)]\}[\tau-(2\omega-1)(1-\alpha)]^{-1} \tag{12c}$$

$$\frac{\dot{\varphi}}{\varphi} = \frac{1}{\psi}-z-\frac{1}{z}\frac{1}{\varphi}+2\rho+(1-\alpha)(1-\tau)A^{1/\alpha}\tau^{1-\alpha/\alpha} \tag{12d}$$

$$\lim_{\to\infty}\left(e^{-\rho t}\frac{\varphi}{\psi}\right)=0 \tag{12e}$$

where $\tau^{@} \equiv (1-\alpha)\frac{\alpha\omega+(1-\alpha)(1-\omega)}{\alpha+(1-\alpha)(1-\omega)}$, $\widetilde{\tau} = 1-\alpha$, $s_1 \equiv (1-\alpha)[2(1-\omega)+\alpha]A^{1/\alpha}$,

$s_2 \equiv \frac{\theta}{2}\left[1+\frac{1-\alpha}{\alpha}(1-\omega)\right]$, $s_3 \equiv 2(1-\alpha)(1-\omega)A^{1/\alpha}$, $s_4 \equiv \frac{1-\alpha}{\alpha+2(1-\omega)}[\alpha+2(1-\omega)(1-\alpha)]$

and $\xi \equiv s_2(1+\alpha\psi z)(\tau-\tau^{@})[s_3\alpha\psi z(\widetilde{\tau}-\tau)+s_1(s_4-\tau)]^{-1}$.

The four unknown variables (τ, z, ψ, and φ) will, hence, be determined at any point of time, in a Symmetric Nash Equilibrium, by equations (12a)-(12d), the initial condition for $\varphi_0=\mu_0 k_0$ and transversality condition (12e).

4.1.1. Long – run equilibrium

Definition 2

Define the long run equilibrium as a Balanced Growth Path where both capital and consumption grow at the same constant positive rate.

From equation (11c) I find out that in order for consumption to grow at a constant rate, $(1-\tau)\frac{y}{k}$ should also be constant. Given that in symmetry $\frac{y}{k}=(A\tau^{1-\alpha})^{\frac{1}{\alpha}}$, I conclude that the tax rate should also be constant in the Balanced Growth Path. Let denote the

Balanced Growth Path values in the Symmetric Long-run Nash Equilibrium (SLNE) of (τ, z, ψ and φ) by ($\tau^\$$, $z^\$$, $\psi^\$$ and $\varphi^\$$). Appendix D proves that in the long run equilibrium it will be:

$$\tau^\$ = (1-\alpha)\frac{\alpha\omega + (1-\alpha)(1-\omega)}{\alpha + (1-\alpha)(1-\omega)} \tag{13a}$$

$$z^\$ = (1-\alpha)(1-\tau^\$)A^{1/\alpha}(\tau^\$)^{1-\alpha/\alpha} + \rho \tag{13b}$$

$$\psi^\$ = \frac{\alpha\omega + (1-\alpha)(1-\omega)}{\alpha(1-\omega)}\frac{1}{z^\$} \tag{13c}$$

$$\varphi^\$ = \left[\frac{\alpha(1-\omega)}{\alpha\omega + (1-\alpha)(1-\omega)}z^\$ + \rho\right]^{-1}\frac{1}{z^\$} \tag{13d}$$

Now I will check whether the resulting Balanced Growth Path solution is well-defined. A well-defined Balanced Growth Path requires: i) $z^\$$ to be positive and grow at a constant rate, ii) the economy to grow, iii) $0<\tau^\$<1$ and iv) transversality condition to hold.

Proposition 2

Condition $\frac{\alpha^2}{1-\alpha}\frac{1+(1-\alpha)(1-2\omega)}{\alpha\omega+(1-\alpha)(1-\omega)}\left[\frac{\alpha\omega+(1-\alpha)(1-\omega)}{\alpha+(1-\alpha)(1-\omega)}\right]^{1/\alpha}[(1-\alpha)A]^{1/\alpha}>\rho+\delta$, *is necessary and sufficient to determine a unique Symmetric Long-run Nash Equilibrium (SLNE) in national policies. This will be summarized by equation (13b) and the tax rate that solves equation (13a), which supports a unique, well-defined Balanced Growth Path, where capital and consumption grow at a constant positive rate, described by equation (13e) below. Intertemporal utility attained will be described by equation (13f).*

$$g^\$ = \alpha(1-\tau^\$)A^{1/\alpha}(\tau^\$)^{1-\alpha/\alpha} - \delta - \rho \tag{13e}$$

$$U^\$ = \frac{1}{\rho^2}[\rho \ln(c_0) + g^\$] + U_0 \tag{13f}$$

Proof: For $\tau^\$=(1-\alpha)\frac{\alpha\omega+(1-\alpha)(1-\omega)}{\alpha+(1-\alpha)(1-\omega)}$ it is obviously $0<\tau^\$<1$. For $z^\$=(1-\alpha)(1-\tau^\$)A^{1/\alpha}(\tau^\$)^{1-\alpha/\alpha}+\rho$, capital's growth rate will be: $g^\$=\left(\frac{\dot{c}}{c}\right)^\$=\left(\frac{\dot{k}}{k}\right)^\$=\alpha(1-\tau^\$)A^{1/\alpha}(\tau^\$)^{1-\alpha/\alpha}-\delta-\rho$. Thus positive growth asks for: $\alpha(1-\tau^\$)A^{1/\alpha}(\tau^\$)^{1-\alpha/\alpha}>\delta+\rho \Leftrightarrow \frac{A}{(\delta+\rho)^\alpha}>\left[\frac{1+(1-\alpha)(1-2\omega)}{(1-\alpha)\alpha^3\omega+\alpha^2(1-\alpha)^2(1-\omega)}\right]^{1-\alpha}\frac{\alpha+(1-\alpha)(1-\omega)}{\alpha^2+\alpha^2(1-\alpha)(1-2\omega)}$. Transversality condition asks for: $\lim_{t\to\infty}\left(e^{-\rho t}\frac{\varphi}{\psi}\right)=0$. Using values found in the Appendix D, I get: $\frac{\varphi^{\$}}{\psi^{\$}}=\frac{\alpha(1-\omega)}{\alpha\omega+(1-\alpha)(1-\omega)}[\frac{\alpha(1-\omega)}{\alpha\omega+(1-\alpha)(1-\omega)}z^{\$}+\rho]^{-1}$, which is constant. Thus the transversality condition will hold. Finally, at the long run equilibrium it will be: $\left(\frac{\dot{c}}{c}\right)^{\$}=g^{\$}$, thus, $c^{\$}=c_0e^{g^{\$}t}$. Using equation (1) one may get intertemporal utility: $\int_0^{\infty}[1n(c^{\$})e^{-\rho t}]dt=\frac{1}{\rho^2}[\rho\ln(c_0)+g^{\$}]+U_0$. *Q.E.D.*

The degree of capital mobility does not affect the long run Nash tax equilibrium, since the Balanced Growth Path values $\tau^{\$}$ and $z^{\$}$ do not depend on the parameter value θ. As capital stock (k) increases, its social value (v) becomes infinitesimal1[18] and the importance of capital mobility diminishes. Note that this result is related to the transversality condition and not to the assumption of symmetry (is still present in an asymmetric framework).

4.1.2 Transitional dynamics

In Appendix E the system (12a)-(12d) is linearized around the Balanced Growth Path described by equations (13a)-(13d). After some algebra one ends up with the following system:

$$\tau=\tau^{\$}+(\tau_0-\tau^{\$})e^{-(\dot{c}/c)^{\$}t} \quad (14a)$$

$$z=z^{\$}+(\tau_0-\tau^{\$})v_{21}e^{-(\dot{c}/c)^{\$}t} \quad (14b)$$

$$\psi=\psi^{\$}+(\tau_0-\tau^{\$})v_{31}e^{-(\dot{c}/c)^{\$}t}+\frac{(\psi^{\$})^2}{\varphi^{\$}}(\frac{1}{\varphi^{\$}}\frac{1}{z^{\$}}-\lambda_3)[(\varphi_0-\varphi^{\$})-(\tau_0-\tau^{\$})v_{41}]e^{\lambda_3 t} \quad (14c)$$

$$\varphi=\varphi^{\$}+(\tau_0-\tau^{\$})v_{41}e^{-(\dot{c}/c)^{\$}t}+[(\varphi_0-\varphi^{\$})-(\tau_0-\tau^{\$})v_{41}]e^{\lambda_3 t} \quad (14d)$$

where v_{21}, v_{31}, v_{41} and λ_3 are constants described in Technical Appendix E.

---

18. Note that $\frac{\varphi}{\psi}=\mu k\frac{v}{\mu}=vk$, thus $v^{\$}k^{\$}=\frac{\varphi^{\$}}{\psi^{\$}}$, which is some constant. In the long run, capital (k) will increase (the economy grows), but its social value (v) will decrease enough to guarantee that vk remains equal to $\frac{\varphi^{\$}}{\psi^{\$}}$. As $k\to\mathbb{N}$, it should be $v\to 0$, i.e. in the long-run it becomes infinitesimal.

Proposition 3

The long run tax rate and the associated equilibrium are locally indeterminate. The degree of capital mobility will not affect economies' Balanced Growth Path in the long run. Solving backwards, to get a value for the Nash tax rate in the first period, proves that this will be affected by the degree of capital mobility. Moreover, through the initial Nash tax rate the degree of capital mobility also affects transitional dynamics.

Proof: See Appendix E

The long run equilibrium proves to be indeterminate, i.e. there are infinite trajectories leading to it. The only predetermined variable in the specific framework is φ – the product of capital stock, k, with the auxiliary variable related to the Euler equation, μ. In the rest of this section, I use φ_0 to determine an initial value for the Nash tax rate. Initial values for the two other variables, ψ_0 and z_0, are also taken into consideration (in order to assure that all related conditions are satisfied), but they are not specifically determined. The specific trajectory economies end up on in the long run equilibrium cannot be pinpointed.
According to equation (14a): $\dot{\tau} = \left(\frac{\dot{c}}{c}\right)^{\$}(\tau^{\$} - \tau_0)e^{-(\dot{c}/c)^{\$}t}$, i.e. τ increases for $\tau_0 < \tau^{\$}$ while it decreases in the opposite case, converging to $\tau^{\$}$ in the long run. At the end of Appendix E one may check that τ_0 depends on a set of parameter values, involving the degree of capital mobility, θ, the productivity of private capital, α, the extent of external contribution of foreign countries' infrastructure to the domestic economy, 1-ω, the level of technology, A, initial values of the private capital stock, k_0, the current value shadow price of households' consumption1[19], μ_0, values at the long run Nash equilibrium for the growth rate[20], $g^{\$}$, the tax rate, $\tau^{\$}$, and, through z_0 and ψ_0, the shadow parameters, $z^{\$}$, $\psi^{\$}$ and $\varphi^{\$}$. Given the non-linear form of equation determining $\tau 0$, it will be more convenient to check whether τ_0 is greater or lower than $\tau^{\$}$ through numerical simulations. For reasonable parameter values numerical result 1 will hold.

Numerical result 1

The Nash tax rate will initially be lower than its long run value as long as the extent of external contribution of foreign countries' public spending to the domestic economy, 1-ω, is clearly lower than the contribution of domestic public spending, ω. The opposite will hold, however, for ω values higher but close to 0.5.

19. Remember that $o \equiv \mu k$, i.e. an initial value for o_0 implies initial values for k_0 and μ_0. Through ψ_0, o_0 determines τ_0.

20. This works through v_{21}, which determines z_0.

Proof: See Appendix F. Note that in order ψ_0 to be well defined, it should be $\varphi_0 > \varphi^{\$} + v_{41}(\tau_0 - \tau) - [\psi^{\$} + v_{31}(\tau_0 - \tau^{\$})]\frac{\varphi^{\$}}{(\psi^{\$})^2}\left(\frac{1}{\varphi^{\$}}\frac{1}{z^{\$}} - \lambda_3\right)^{-1}$. *Q.E.D.*

Equation (11a) may also be written as: $\frac{1}{2}\left[1 + \frac{1-\alpha}{\alpha}(1-\omega)\right](1+\alpha\psi z)[(1-\alpha)\frac{\alpha\omega + (1-\alpha)(1-\omega)}{\alpha + (1-\alpha)(1-\omega)} - \tau] + \frac{1}{\theta}(1-\tau)(1-\alpha)\frac{1}{k}\frac{y}{k}\ \{2(1-\omega)(1-\alpha-\tau)(1+\alpha\psi z) + \alpha[(1-\alpha)(2\omega-1) - \tau]\} = 0$ In the absence of capital mobility (i.e. for $\theta \to \infty$) it asks for: $\tau^* = (1-\alpha)\frac{\alpha\omega + (1-\alpha)(1-\omega)}{\alpha + (1-\alpha)(1-\omega)}$, i.e. it will be equal to the long run Nash rate in the presence of capital mobility1[21]. Thus, numerical result 1 predicts that, in most of the cases, in the presence of capital mobility the Nash tax rate will initially be lower than the Nash tax rate in the absence of it. In the special (rather not realistic) case where the extent of external contribution of foreign countries' infrastructure to the domestic economy is almost equal to the contribution of domestic infrastructure, the Nash tax rate in the presence of capital mobility will initially be higher than the Nash tax rate in the absence of it. The rationale of this finding is related to the work of two opposite effects.

Capital mobility initiates a 'race to the bottom' – a rather famous result in the literature1[22]. Note that for $\tau > (1-\alpha)(2\omega-1)$2[23] in symmetry $\frac{\partial k_j}{\partial \tau_j} < 0$, i.e. by lowering the tax rate, private capital stock will increase. A higher tax rate has a negative direct effect on the domestic interest rate, while it will not affect directly the interest rate abroad. On the other hand, as Sorensen (2000, p. 439) notes, "by raising spending on infrastructure, a government can attract mobile capital because a better infrastructure increases the profitability of domestic investment". The former effect counterweights the latter for $\tau > (1-\alpha)(2\omega-1)$. Thus, governments have an incentive to choose a lower tax rate in order to attract capital – an observation similar to the one made by Noiset (1995).

The second effect at work here is probably related to what Persson and Tabellini (1992, p. 694) call the 'tax the foreigner' effect, which "tends to push the tax rate on capital above the Pareto efficient frontier". A rise in the domestic tax will have a

21. This result is also found in the running working paper (Savvidis H. "International positive production externalities under a transfer payment scheme – the case for cooperation") examining a similar framework without capital mobility but with the addition of transfer payments between governments.

22. For an insightful model see Persson and Tabellini (1995)

23. Through numerical simulations one may check that this will hold for reasonable parameter values.

direct negative impact on the after tax income of foreign owners, i.e. governments may 'export' taxation. Foreign investors will share a burden of the increase in tax revenues, giving government an incentive to choose higher tax rate.

Overall, the 'race to the bottom' effect will usually prevail, resulting in a lower Nash tax rate. For ω values close to 0.5, however, governments may end up choosing a higher Nash tax rate, with the latter effect being stronger than the former. The rationale is that for ω values close to 0.5, the relative importance of the capital flight decreases, since production abroad has a strong impact (through foreign infrastructure) on the domestic economy. Yet, this is a rather extreme case, since it actually assumes e.g. that a road in Athens affects equally production in Greece and Germany.

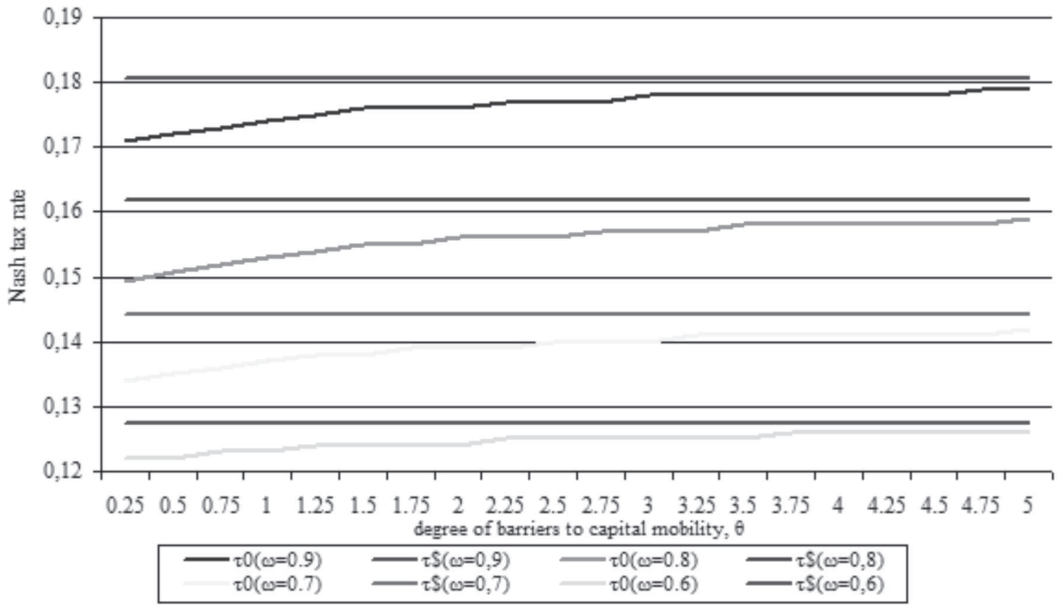
4.1.3 Comparative dynamics

According to equation (13a), the Nash tax rate in the Balanced Growth Path will be equal to the Nash tax rate in the case of no capital mobility, depending on ω and α. In the transitional path the Nash tax rate will be described by equation (14a), depending on the Balanced Growth Path Nash tax rate, $\tau^{\$}$, the Balanced Growth Path growth rate, $g^{\$}$, and the initially chosen Nash tax rate, τ_0. The latter depends on several parameter values, including θ, the degree of capital mobility. Note that equation (11a) is the only one where the specific parameter is present. The degree of capital mobility will directly dictate only the initial Nash tax rate, τ_0 and through it will also affect the rest variables of the dynamic system. In the long run, however, it will not have any influence.

Figures1[24] 1a – 1b describe the relationship between τ_0 and θ, for different values of ω. In figure 1a, the positive sloping lines describe how, for a given value of ω (0.6 or 0.7 or 0.8 or 0.9), the initial Nash tax rate, τ_0, increases as the degree of obstacles to capital mobility, θ increases as well (or as the degree of capital mobility decreases). In the same figure, horizontal lines depict the level of the long run Nash tax rate for different values of ω. As the degree of capital mobility decreases (i.e. as θ increases), the difference between the initial and the long run Nash tax rate decreases.

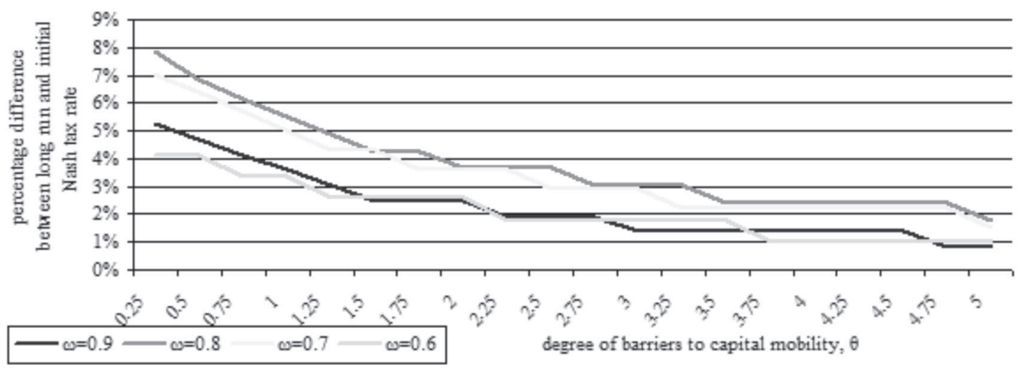
24. In all figures it is: α=0.8, k_0=1, $\rho=\delta$=3% and $g^{\$}$=2.5%. Relationships described above, however, hold also for other parameter values.

Figure 1a. Initial and long run Nash tax rate for different degrees of capital mobility and levels of ω significantly higher than 0.5



This is also obvious in figure 1b, where each line describes how the percentage difference between the initial and the long run Nash tax rate, $\frac{\tau^{\$} - \tau_0}{\tau^{\$}}$, changes as the degree of obstacles to capital mobility increases, for a given value of ω. For all ω values the percentage difference deteriorates as the degree of capital mobility decreases (i.e. as θ increases). Moreover there is no significant variation in the percentage difference as ω changes.

Figure 1b. The percentage difference between the initial and the long run Nash tax rate for different degrees of capital mobility and levels of ω significantly higher than 0.5



Figures 2a-2b describe the relationship between τ_0 and θ, for values of ω close to 0.5. In figure 2a one may check that for ω=0.53 the initial Nash tax rate, τ0, increases as the degree of capital mobility decreases (or as θ increases), converging to its long run value, which is higher than the initial value. The same stands also for ω=0.52, but the initial Nash tax rate differs even less than its long run value. On the other hand, for ω=0.51 the initial Nash tax rate is higher than its long run value, converging to it (i.e. decreasing) as the degree of capital mobility decreases (or as θ increases). For ω=0.5, the difference between the initial and the long run Nash tax rate increases. In figure 2b the four lines describe how the percentage difference between the initial and the long run Nash tax rate, $\frac{\tau^{\$} - \tau_0}{\tau^{\$}}$, changes as the degree of obstacles to capital mobility increases, for a given value of ω. For ω=0.53 and ω=0.52 the difference is positive and decreases with θ, while for ω=0.51 and ω=0.5 it is negative and increases with θ.

Figure 2a. Initial and long run Nash tax rate for different degrees of capital mobility and levels of ω close to 0.5

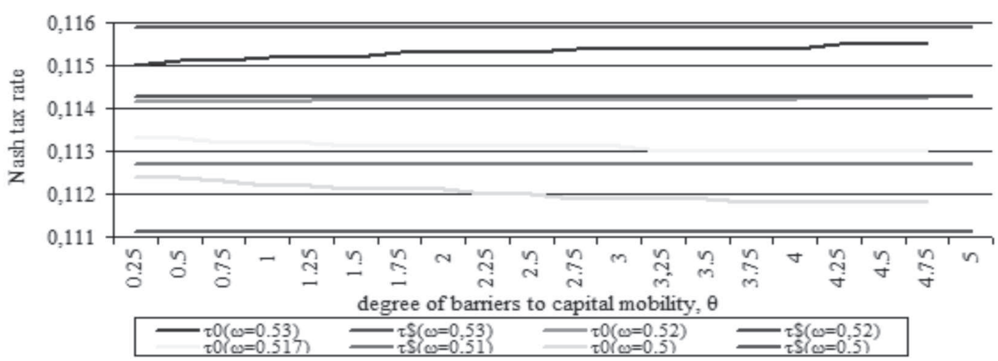
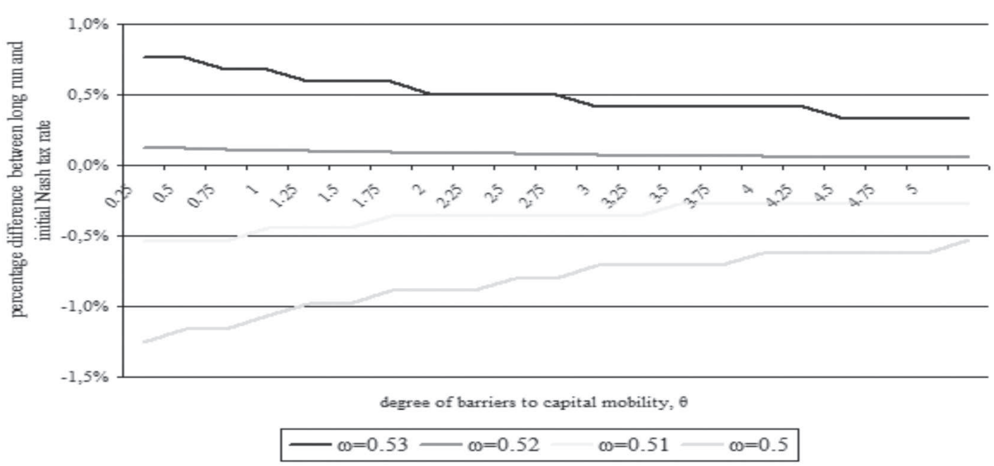


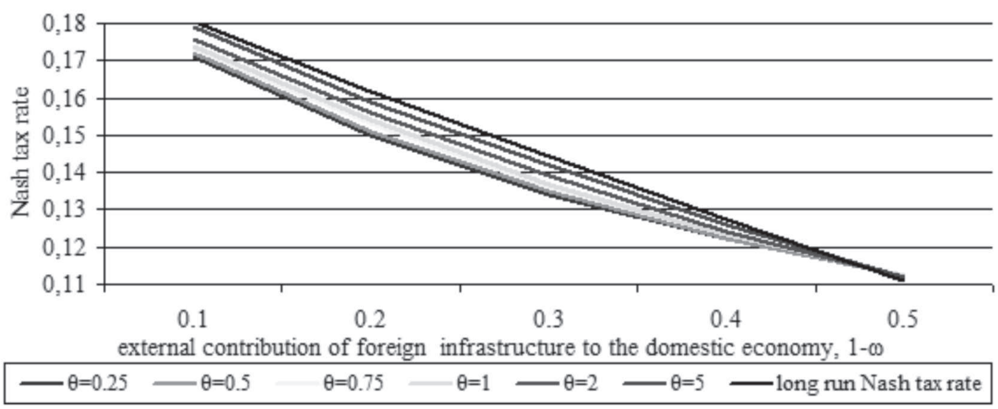
Figure 2b. The percentage difference between initial and long run Nash tax rate for different degrees of capital mobility and levels of ω close to 0.5



The rationale of the above findings is that as capital mobility obstacles increase, the initial Nash tax rate converges to the value it has in the case of no capital mobility (which is equal to the long run Nash tax rate when private capital is mobile), with the 'race to the bottom' effect (or the prevailing 'tax the foreigner' effect in the extreme case of ω close to 0.5) diminishing. On the other hand, as private capital becomes more mobile, the 'race to the bottom' effect (or the 'tax the foreigner' effect) intensifies.

The initial Nash tax rate, τ_0, decreases as the extent of external contribution of foreign countries' infrastructure to the domestic economy, 1-ω, increases. Indeed in figure 3 we see that, no matter the degree of capital mobility, θ, the initial Nash tax rate, τ_0, decreases as foreign infrastructure becomes more important for the domestic economy, i.e. as 1-ω increases. In the same figure one may also see that the long run Nash tax rate (which does not depend on the degree of capital mobility) also decreases with 1-ω. Again, the difference between the initial Nash tax rate and its long run value (the black line) increases for higher θ values, while for 1-ω close to 0.5 the initial Nash tax rate is higher than its long run value.

Figure 3. Initial and long run Nash tax rate for different levels of external contribution of foreign infrastructure to the domestic economy



Numerical Result 2

The initial Nash tax rate, τ_0, differs more from its long run value, as private capital is more mobile. Thus, as long as it is lower than it, it increases with θ, while in case initial Nash tax rate, τ_0, is higher than the long run Nash tax rate, $\tau^{\$}$ (for ω values close to 0.5) it decreases with θ. Moreover the initial Nash tax rate, τ0, depends negatively on the extent of foreign public spending contribution to the domestic economy, 1-ω.

Proof: See results of numerical simulations presented in figures above. Q.E.D.

Cooperative national policies

Consider now the benchmark case, where all governments cooperate to choose a common tax rate, τ, in order to intertemporally maximize utility for both countries, given economies' budget constraint and decisions of private sector in each economy. The maximization problem will be: $\max_{\tau}\int_0^{\infty}[\ln(c_j)e^{-\rho t}+\ln(c_f)e^{-\rho t}]dt$, s.t. equations of the WDCE (equations (2) - (9) above). The focus will be on Symmetric Cooperative Solutions in national policy rules. That is, economies may differ *ex ante* but they become identical *ex post*. Invoking symmetry into the FOC's, after some algebra (shown in Appendix G) one ends up with Proposition 4, below.

Proposition 4

A Symmetric Cooperative Solution (SCS) in national tax policies is summarized by equations (15a)-(15f) below. Equation (15a) determines a unique tax rate, denoted as $0<\tilde{\tau}<1$. *Cooperative national policies are not affected at all by the degree of capital mobility.*

Proof: See Appendix G. There will also be the Euler equation (15b), the budget constraint (15c) and the transversality condition (15f). Note that in equations (15a)-(15f) the coefficient of capital mobility, θ, is absent. Q.E.D.

$$\tilde{\tau}=1-\alpha \tag{15a}$$

$$\frac{\dot{c}}{c}=\alpha(1-\tau)\frac{y}{k}-\delta-\rho \tag{15b}$$

$$\frac{\dot{k}}{k}=(1-\tau)\frac{y}{k}-\delta-\frac{c}{k} \tag{15c}$$

$$\frac{\dot{v}}{v}=\rho+\delta-(1-\tau)\frac{y}{k} \tag{15d}$$

$$\frac{\dot{\mu}}{\mu}=\frac{v}{\mu}-\frac{1}{c}\frac{1}{\mu}-\alpha(1-\tau)\frac{y}{k}+\delta+2\rho \tag{15e}$$

$$\lim_{t\to\infty}(kve^{-\rho t})=0 \tag{15f}$$

In this case, the optimal tax rate is independent of the degree of capital mobility, θ, and the external contribution of foreign countries' infrastructure to the domestic economy, 1-ω. Optimal tax rate is just equal to the rate of public factor's productivity, 1-α. This is actually Barro's (1990) well-known solution for the tax rate. The optimal tax rate is non-state contingent, i.e. it doesn't change over time. This result, however, is not universal, coming from the specific form adopted for the production function (Cobb-Douglas) and economy's budget constraint (linear).

Let denote the Balanced Growth Path values in the Symmetric Long-run Cooperative Solution (SLCS) of (τ, c, k) by $(\tilde{\tau}, \tilde{c}, \tilde{k})$. According to Definition 2 a well-defined Balanced Growth Path requires: i) $\tilde{c}$ & $\tilde{k}$ to be positive and grow at the same rate, ii) the economy to grow, iii) $0<\tilde{\tau}<1$ and iv) transversality condition to hold.

After some algebra (see Appendix H), the dynamic system described above generates the following equations:

$$\tilde{\tau}=1-\alpha \tag{15a}$$

$$\tilde{c}=k_0[(1-\alpha)(1-\tau)(A\tilde{\tau}^{1-\alpha})^{\frac{1}{\alpha}}+\rho]e^{\left[\alpha(1-\tilde{\tau})(A\tilde{\tau}^{1-\alpha})^{1/\alpha}-\delta-\rho\right]t} \tag{16a}$$

$$\tilde{k}=k_0e^{\left[\alpha(1-\tilde{\tau})(A\tilde{\tau}^{1-\alpha})^{1/\alpha}-\delta-\rho\right]t} \tag{16b}$$

As it is also shown in Appendix H, for this system it will always be: $\frac{\tilde{c}}{\tilde{k}}=(1-\alpha)\,(1-\tilde{\tau})(A\tilde{\tau}^{1-\alpha})^{\frac{1}{\alpha}}+\rho$, i.e. there will be no transitional dynamics, as in all AK models. Economies either start on the Balanced Growth Path or 'jump' to it.

Proposition 5

Condition $\alpha^2A^{1/\alpha}(1-\alpha)^{1-\alpha/\alpha}>\delta+\rho$ is necessary and sufficient to determine a Symmetric Long-run Cooperative Solution (SLCS) in national policies. This will be summarized by equations (16a) and (16b) and the tax rate solving equation (15a), which is not affected by the degree of capital mobility. This tax rate supports a unique, well-defined balanced growth path, in which capital and consumption grow at a constant positive rate, described by equation (17a) below.

Proof: See Appendix H

$$\tilde{g}=\alpha(1-\tilde{\tau})A^{1/\alpha}\tilde{\tau}^{1-\alpha/\alpha}-\rho-\delta \tag{17a}$$

Combining equations (17a) and (1) one gets:

$$\tilde{U}=\frac{1}{\rho^2}[\rho\ln(c_0)+\tilde{g}]+U_0 \tag{17b}$$

5. Non-cooperative versus cooperative outcome

When governments cooperate, they internalise the externality created by the effect public spending in economy f has on economy j's production function. This is actually the incentive governments have to cooperate, as declared by Proposition 6, below.

Proposition 6

The optimal tax rate and the associated growth rate in Symmetric Long-run Cooperative Solution (SLCS) will be higher than the optimal tax rate and the associated growth rate in Symmetric Long-run Nash Equilibrium (SLNE), $\tilde{\tau} > \tau^{\$}$*. At SLCS utility attained will always be higher than in the case of SLNE. The merits of cooperation will increase, as the external contribution of foreign countries' public spending to the domestic economy, 1–ω, intensifies.*

Proof: It is $\tilde{\tau} - \tau^{\$} = \dfrac{\alpha(1-\alpha)(1-\omega)}{\alpha + (1-\alpha)(1-\omega)} > 0 \Leftrightarrow \tilde{\tau} > \tau^{\$}$, where $\dfrac{\partial(\tilde{\tau} - \tau^{\$})}{\partial(1-\omega)} =$

$= \dfrac{(1-\alpha)\alpha^2}{[\alpha + (1-\alpha)(1-\omega)]^2} \Leftrightarrow \dfrac{\partial(\tilde{\tau} - \tau^{\$})}{\partial(1-\omega)} > 0$. Also check that at the long run equilibrium both and $g^{\$}$ depend positively on the tax rate ($\tilde{\tau}$ and $\tau^{\$}$, respectively) as long as it is lower than 1-α (i.e. the productivity of G): $\dfrac{\partial\tilde{g}}{\partial\tilde{\tau}} = A^{1/\alpha}\tilde{\tau}^{1-\alpha/\alpha}\dfrac{1-\alpha-\tilde{\tau}}{\tilde{\tau}}$ and $\dfrac{\partial g^{\$}}{\partial\tau^{\$}} = A^{1/\alpha}(\tau^{\$})^{1-\alpha/\alpha}\dfrac{1-\alpha-\tau^{\$}}{\tau^{\$}}$. Given that $\tilde{\tau} = 1-\alpha > \tau^{\$}=1\text{-}\alpha>\tau^{\$}$ one may conclude that $\tilde{g} > g^{\$} > g^{\$}$. From equations (13f) and (17b) one gets: $\tilde{U} - U^{\$} = \dfrac{1}{\rho^2}(\tilde{g} - g^{\$}) > 0$, i.e. $\tilde{U} > U^{\$}$. Finally, check that $\dfrac{\partial\tilde{g}}{\partial\omega}=0$, while $\dfrac{\partial g^{\$}}{\partial\omega} = \dfrac{\partial\tilde{g}}{\partial\tau^{\$}}\dfrac{\partial\tau^{\$}}{\partial\omega} > 0$, since $\dfrac{\partial\tau^{\$}}{\partial\omega}$ $=(1-\alpha)\left[\dfrac{\alpha}{\alpha + (1-\alpha)(1-\omega)}\right]^2 > 0$ and $\tau^{\$} < 1-\alpha$. Thus, $\dfrac{\partial\tilde{g}}{\partial\omega} - \dfrac{\partial g^{\$}}{\partial\omega} < 0$ and $\dfrac{\partial(\tilde{U} - U^{\$})}{\partial(1-\omega)} = \dfrac{1}{\rho^2}\dfrac{\partial(\tilde{g} - g^{\$})}{\partial(1-\omega)} > 0$.

Q.E.D.

On top of the 'free riding' effect arising from the existence of the common public good in countries' production function, there are also two other effects at work: the 'race to the bottom' effect and the 'tax the foreigner' effect. As explained above (section 3), while the former results in a lower Nash tax rate, the latter pushes it in the opposite direction. For reasonable parameter values the 'race to the bottom' will prevail, resulting in a lower Nash tax rate in the presence of capital mobility.

In the special case of 1-ω being close to 0.5, the Nash tax rate in the presence of capital mobility will initially be higher than the Nash tax rate chosen in the absence of it (but never higher than the cooperative solution). Only then, capital mobility may serve as a partial substitute of cooperation. Even in that case, however, capital mobility does not affect the tax rate in the long run: Both 'race to the bottom' and 'tax the foreigner' effects fade out, while the 'free riding effect' still works, resulting in a Nash tax rate (whether capital is mobile or not) lower than the cooperative solution (Proposition 6). The two effects diminish, because capital's social value (v) becomes infinitesimal in the long run, as already discussed. Governments' incentive to attract capital from neighboring economies by lowering the tax rate or to "export" taxation by increasing it becomes insignificant, while the incentive to "free ride" foreign public services remains intact.

Proposition 7

In the presence of capital mobility, the tax rate chosen when governments play Nash to each other will always be lower than the one chosen when they cooperate.

Proof: In Appendix E. it is shown that initial Nash tax rate, τ_0, will be described by equation: $\frac{\theta}{2}[1+\frac{1-\alpha}{\alpha}(1-\omega)](1+\alpha z_0\psi_0)(\tau_0-\tau^{\$})=(1-\tau_0)(1-\alpha)\frac{1}{k_0}(A\tau_0^{1-\alpha})^{1/\alpha}\{2(1-\omega)(1-\alpha-\tau_0)(1+\alpha z_0\psi_0)+\alpha[(1-\alpha)(2\omega-1)-\tau_0]\}$. Let $\tau_0 \geq \tilde{\tau}$. According to eq. (15a) and Proposition 6, it is: $\tilde{\tau}=1-\alpha>\tau^{\$}$, thus it will also be: $\tau_0>\tau^{\$}$. The left hand side of the equation determining τ_0 will, thus, be positive and given that $(1-\tau_0)(1-\alpha)\frac{1}{k_0}(A\tau_0^{1-\alpha})^{1/\alpha}>0$, expression $2(1-\omega)(1-\alpha-\tau_0)(1+\alpha z_0\psi_0)+\alpha[(1-\alpha)(2\omega-1)-\tau_0]$ should be positive as well. It is $\omega<1\Leftrightarrow 2\omega-1<1$. For $\tau_0\geq 1-\alpha$ it should also be: $\tau_0>(1-\alpha)(2\omega-1)\Leftrightarrow\alpha[(1-\alpha)(2\omega-1)-\tau_0]<0$. However, it will also be: $2(1-\omega)(1-\alpha-\tau_0)(1+\alpha z_0\psi_0)\leq 0$, thus the right hand side of the expression will not be positive. Given that Nash tax rate will converge to $\tau^{\$}$, which is lower than $\tilde{\tau}$ (Proposition 6), the tax rate when governments play Nash to each other will always be lower than $\tilde{\tau}$. *Q.E.D.*

The degree of capital mobility, θ, affects transitional dynamics when governments play Nash to each other, but not when they cooperate. Thus, θ affects the difference between the tax rate governments choose when they cooperate and the one they choose when they do not during transitional dynamics.

Numerical result 3

For reasonable parameter values, the difference between the initially chosen Nash tax rate and the cooperative solution depends positively on the degree of capital mobility (i.e. it depends negatively to θ). The opposite will hold, for ω values higher but close to 0.5. In the long run, however, the above difference is independent of the degree of capital mobility.

Proof: As proved above, when governments cooperate there are no transitional dynamics. Economies either start on or 'jump' to the Balanced Growth Path, where the degree of capital mobility, θ, does not have any effect. On the contrary, the chosen tax rate when governments play Nash to each other depends on θ as it converges to its long run value. According to numerical results 2, the initial Nash tax rate, τ_0, differs more from its long run value, as private capital is more mobile (i.e. for lower θ values). Thus, as long as it is lower than it, it increases with θ, while in case initial Nash tax rate, τ_0, is higher than the long run Nash tax rate, $\tau^{\$}$ (for ω values higher but close to 0.5 – see numerical result 1) it decreases with θ. *Q.E.D.*

The intuition behind these findings is that as capital becomes more mobile, ‘race to the bottom’ and ‘tax the foreigner’ effects intensify. In the case of reasonable parameter values, the first effect prevails, resulting in a lower tax rate. In the extreme case of ω higher but close to 0.5, however, the latter effect prevails, resulting in a higher tax rate, which is still lower than the one chosen when governments cooperate (Proposition 7).

6. Conclusion

Previous sections demonstrate that, in the presence of capital mobility, governments have an incentive to cooperate in setting tax policy, when economies are interlinked through some positive production spillover effect. Moreover, the merits of cooperation will increase as the external contribution of foreign countries’ public spending to the domestic economy intensifies (Proposition 6). As Cooper and John (1988) proved in their seminal paper, in the presence of positive spillovers, players’ actions increase when switching from an uncoordinated to a coordinated equilibrium. Allowing capitalists to invest abroad (i.e. capital mobility) doesn’t destroy the incentive to cooperate, since it does not affect optimal policies in the symmetric long run equilibrium. As economies grow and capital stock increases, its social value diminishes and the degree of capital mobility does not affect optimal policies. It affects, however, transitional dynamics when governments play Nash to each other.

When capital is allowed to move between economies, governments start with a tax rate different to the one chosen in the absence of capital mobility. That difference depends positively on the degree of capital mobility (numerical result 2) and, for reasonable parameter values (ω clearly higher than 0.5), it results to a tax rate even lower than the one in the Symmetric Long-run Nash Equilibrium (numerical result 1). The intuition is that, in the short run, capital mobility triggers a ‘race to the bottom’ effect, which comes in addition to the ‘free riding’ effect existing in the specific outline (because of positive international production spillovers). Allowing for capital mobility does not work as an alternative to cooperation - on the contrary it intensifies the need to cooperate.

This argument stands, obviously, only for the specific framework studied here. It may, in no way, be understood as a general result against capital mobility. Even if one allows for international production externalities, capital mobility may have utility enhancing effects for specific economic groups, e.g. by affecting the net return to private capital. In the present framework households own both private capital and labour. By adopting a more sophisticated framework one may allow for different economic groups (e.g. capitalists and workers) and study how capital mobility affects their net income under either benevolent or partisan governments.

7. Appendix

A. Economy's budget constraint

Per capita net return for agent h of country j for his foreign investment may be written as: $\frac{R_{hj,f}}{L_{hj}}=b_{hj,f}(r_f\frac{\theta}{2}-b_{hj,f})=\frac{r_f-r_j}{\theta}\left(r_f-\frac{\theta}{2}\frac{r_f-r_j}{\theta}\right)=\frac{r_f-r_j}{\theta}\frac{r_f+r_j}{2}=\frac{r_f^2-r_j^2}{2\theta}$. Also remember that: $b_{hj}=b_{hj,j}+b_{hj,f}\Leftrightarrow b_{hj,j}=b_{hj}-b_{hj,f}=b_{hj}-\frac{r_f-r_j}{\theta}$, hence private agent's budget constraint becomes: $c_{hj}+\dot{b}_{hj}=w_j+r_j(b_{hj}-\frac{r_f-r_j}{\theta})+\frac{r_f^2-r_j^2}{2\theta}=w_j+r_jb_{hj}+\frac{(r_f-r_j)^2}{2\theta}$. Given normalization of labor force to unity it will be $b_j=b_{hj}$. At the economy level one gets: $\dot{b}_j=w_j+r_jb_j+\frac{(r_f-r_j)^2}{2\theta}-c_j$.

B. Symmetric Nash Equilibrium

Utility maximization by the government in country j results in a current value Hamiltonian of the form: $J_j=\ln(c_j)+\mu_jc_j(r_j-\rho)+v_j[w_j+b_jr_j-c_j+\frac{(r_f-r_j)^2}{2\theta}]$, where μ_j and v_j are multipliers associated with equations (4) & (7), respectively. The first-order conditions (FOC's) with respect to τ_j, k_j and c_j may now be derived.

Optimization asks for: $\frac{\partial J_j}{\partial\tau_j}=0\Leftrightarrow\mu_jc_j\frac{\partial r_j}{\partial\tau_j}+v_j[\frac{\partial w_j}{\partial\tau_j}+b_j\frac{\partial r_j}{\partial\tau_j}+\frac{r_f-r_j}{\theta}(\frac{\partial r_f}{\partial\tau_j}-\frac{\partial r_j}{\partial\tau_j})]=0$. In Technical Appendix A it is shown that, after some algebra, one ends up with: $\frac{\theta}{2}[1+\frac{1-\alpha}{\alpha}(1-\omega)](vk+\alpha\mu c)[(1-\alpha)\frac{\alpha\omega+(1-\alpha)(1-\omega)}{\alpha+(1-\alpha)(1-\omega)}-\tau]+(1-\tau)(1-\alpha)\frac{1}{k}\frac{y}{k}\{2(1-\omega)(1-\alpha-\tau)(vk+\alpha\mu c)+\alpha vk[(1-\alpha)(2\omega-1)-\tau]\}=0$.

Remember that $k_j=b_j+\frac{\theta}{2}(r_j-r_f)$. In symmetry $r_j=r_f$ and thus $k_j=b_j$. Thus, it will be: $\dot{k}_j=\dot{b}_j=w_j+r_jk_j-c_j$.

Euler equation should also hold: $\frac{\dot{c}}{c}=\alpha(1-\tau)\frac{y}{k}-\delta-\rho$.

Turn to the second First Order Condition, which asks for: $\frac{\partial J_j}{\partial b_j}=\rho v_j-\dot{v}_j$. Through some algebra (Technical Appendix B) one ends up with: $\frac{\dot{v}}{v}=\rho-r-\frac{y}{k}(1-\alpha)(1-\tau)\{\frac{\theta}{2}[\alpha\omega+(1-\alpha)$

$(1\text{-}\omega)]+2\alpha(1\text{-}\alpha)(1\text{-}\tau)(1\text{-}\omega)\frac{y}{k}\frac{1}{k}\text{-}\alpha\frac{\mu}{v}\frac{c}{k}(1\text{-}\omega)\frac{\theta}{2}\}\{\frac{\theta}{2}[1\text{-}(1\text{-}\alpha)(2\omega\text{-}1)]+4\alpha(1\text{-}\alpha)(1\text{-}\tau)$ $(1\text{-}\omega)\frac{y}{k}\frac{1}{k}\}^{-1}$.

The last FOC in symmetry yields to: $\frac{\dot{\mu}}{\mu}=\frac{v}{\mu}-\frac{1}{c}\frac{1}{\mu}\text{-}[\alpha(1\text{-}\tau)\frac{y}{k}\text{-}\delta\text{-}2\rho]$. Finally, the transversality condition should also be satisfied: $\lim_{t\to\infty}(e^{-\rho t}vk)=0$.

C. Variables' transformation

Given that vk≠0, equation (11a) may be written as: $\frac{\theta}{2}[1+\frac{1-\alpha}{\alpha}(1\text{-}\omega)](1+\alpha\psi z)$ $[(1\text{-}\alpha)\frac{\alpha\omega+(1-\alpha)(1-\omega)}{\alpha+(1-\alpha)(1-\omega)}\text{-}\tau]+(1\text{-}\tau)(1\text{-}\alpha)\frac{1}{k}\frac{y}{k}\{2(1\text{-}\omega)(1\text{-}\alpha\text{-}\tau)(1+\alpha\psi z)+\alpha[(1\text{-}\alpha)$ $(2\omega\text{-}1)\text{-}\tau]\}=0$. After some algebra (Technical Appendix C) one gets: $\{[z+$ $\frac{1}{\psi}-\frac{1}{z}\frac{1}{\varphi}-\frac{1}{\alpha}\frac{1}{\tau-(2\omega-1)(1-\alpha)}(1\text{-}\alpha)(1\text{-}\tau)(1\text{-}\omega)(1+\alpha\psi z)(1\text{-}\alpha\text{-}\tau)(A\tau^{1-\alpha})^{1/\alpha}]\frac{1}{\xi}[s_2(\tau^{@}\text{-}$ $\tau)+\xi s_3(\tilde{\tau}\text{-}\tau)][\xi s_1(s_4\text{-}\tau)\text{-}s_2(\tau\text{-}\tau^{@})]+[(1\text{-}\tau)(A\tau^{1-\alpha})^{1/\alpha}\text{-}\delta\text{-}z]s_2(\tau^{@}\text{-}\tau)[s_1(s_4\text{-}\tau)+s_3(\tau\text{-}\tilde{\tau})]\}$ $\{s_2\{[s_3(\tau\text{-}\tilde{\tau})+s_1(s_4\text{-}\tau)][1\text{-}\tau^{@}\text{-}(\tau\text{-}\tau^{@})\frac{1-\alpha}{\alpha}\frac{1-\tau}{\tau}]\text{-}(1\text{-}\tau)(\tau\text{-}\tau^{@})(s_3\text{-}s_1)\}+s_1s_3\,\xi(1\text{-}\tau)(s_4\text{-}\tilde{\tau})\}^{-1}$, where $\tau^{@}$, $\tilde{\tau}$, s_1, s_2, s_3, s_4 and ξ are defined in Technical Appendix C.

From eq. (11e) and (11d) one gets: $\frac{\dot{\psi}}{\psi}=\frac{\dot{\mu}}{\mu}-\frac{\dot{v}}{v}$, which after some algebra (again Technical Appendix C) becomes: $\frac{\dot{\psi}}{\psi}=\frac{1}{\psi}=+\rho\text{-}\frac{1}{z}\frac{1}{\varphi}+(A\tau)^{1/\alpha}\frac{1-\tau}{\tau}\frac{1-\alpha}{\alpha}\{\tau[\alpha+(1\text{-}\omega)$ $(1+\alpha\psi z)]\text{-}(1\text{-}\alpha)[\alpha(2\omega\text{-}1)+(1\text{-}\omega)(1+\alpha\psi z)]\}[\tau\text{-}(2\omega\text{-}1)(1\text{-}\alpha)]^{-1}$.

From eq. (11b) and (11e) one gets: $\frac{\dot{\varphi}}{\varphi}=\frac{\dot{\mu}}{\mu}+\frac{\dot{k}}{k}=\frac{1}{\psi}-\frac{1}{z}\frac{1}{\varphi}+2\rho+(1\text{-}\alpha)(1\text{-}\tau)\frac{y}{k}\text{-}z$.

Also, from eq. (11c) and (11b) one gets: $\frac{\dot{z}}{z}=\frac{\dot{c}}{c}-\frac{\dot{k}}{k}\Leftrightarrow\frac{\dot{z}}{z}=z\text{-}(1\text{-}\alpha)(1\text{-}\tau)\frac{y}{k}\text{-}\rho$.

Finally, the transversality condition may be written as: $\lim_{t\to\infty}\left(e^{-\rho t}\frac{vk\mu}{\mu}\right)=\lim_{t\to\infty}\left(e^{-\rho t}\frac{\varphi}{\psi}\right)=0$.

D. Symmetric Long-run Nash Equilibrium

From equation (12a) one gets, after some algebra (Technical Appendix D):

$\tau^{\$}=(1\text{-}\alpha)\frac{\alpha\omega+(1-\alpha)(1-\omega)}{\alpha+(1-\alpha)(1-\omega)}$.

At the Balanced Growth Path it should be: $\frac{\dot{z}}{z}=0\Leftrightarrow z^{\$}\text{-}(1\text{-}\alpha)(1\text{-}\tau^{\$})\left(\frac{y}{k}\right)^{\$}\text{-}\rho=0$ and given that $\left(\frac{y}{k}\right)^{\$}=[(\tau^{\$})^{1-\alpha}A]^{\frac{1}{\alpha}}$ one ends up with: $z^{\$}=(1\text{-}\alpha)(1\text{-}\tau^{\$})A^{1/\alpha}(\tau^{\$})^{1-\alpha/\alpha}+\rho>0$.

The growth rate should be positive, thus it will be: $\lim_{t\to\infty}\left(\frac{1}{k}\right)=0$. Given that $\dot{\tau}=0$ at the Balanced Growth Path, it will also be: $\lim_{t\to\infty}\left(\frac{\partial RHS}{\partial t}\right)=0$, where RHS is defined in Technical Appendix C. It is $\frac{\partial RHS}{\partial t}=\left(\frac{\dot{z}}{z}+\frac{\dot{\psi}}{\psi}\right)RHS$. Given that $\frac{\dot{z}}{z}=0$ at the Balanced Growth Path, it should also be $\frac{\dot{\psi}}{\psi}=0$. In that case, at the Balanced Growth Path it is also $\frac{\dot{\varphi}}{\varphi}=0$. For $\frac{\dot{\varphi}}{\varphi}=0$ it will be: $\frac{1}{\psi^{\$}}-z^{\$}-\frac{1}{z^{\$}}\frac{1}{\varphi^{\$}}+2\rho+(1-\alpha)(1-\tau^{\$})\left(\frac{y}{k}\right)^{\$}=0 \Leftrightarrow \frac{1}{z^{\$}}\frac{1}{\varphi^{\$}}=\frac{1}{\psi^{\$}}+\rho$. Moreover: $\frac{\dot{\psi}}{\psi}=0 \Leftrightarrow (1-\omega)(1+\alpha\psi^{\$}z^{\$})(1-\alpha-\tau^{\$})=\alpha[\tau^{\$}-(2\omega-1)(1-\alpha)]$, which for $\tau=\tau^{\$}$ yields to: $\psi^{\$}z^{\$}=[\tau^{\$}-(2\omega-1)(1-\alpha)](1-\omega)^{-1}(1-\alpha-\tau^{\$})^{-1}-\frac{1}{\alpha} \Leftrightarrow \psi^{\$}=\frac{\alpha\omega+(1-\alpha)(1-\omega)}{\alpha(1-\omega)}\frac{1}{z^{\$}}$ and $\frac{1}{z^{\$}}\frac{1}{\varphi^{\$}}=\frac{1}{\psi^{\$}}+\rho \Leftrightarrow \varphi^{\$}=[\frac{\alpha(1-\omega)}{\alpha\omega+(1-\alpha)(1-\omega)}z^{\$}+\rho]^{-1}\frac{1}{z^{\$}}$.

E. Linearization around the Balanced Growth Path

In Technical Appendix E, I use Taylor approximation to linearize the system described by equations (12a)-(12d) around the Balanced Growth Path described by equations (13a)-(13d).

It proves to be the case that there are two positive and two negative eigenvalues. The auxiliary variable φ will be the only predetermined value. According to Blanchard and Kahn (1980) if the number of the positive eigenvalues is less than the number of non-predetermined variables (which is the case here), there is infinity of solutions, i.e. local indeterminacy.

Linearization yields to:

$$\tau=\tau^{\$}+(\tau_0-\tau^{\$})e^{-(\dot{c}/c)^{\$}t}$$

$$z=z^{\$}+(\tau_0-\tau^{\$})v_{21}e^{-(\dot{c}/c)^{\$}t}$$

$$\psi=\psi^{\$}+(\tau_0-\tau^{\$})v_{31}e^{-(\dot{c}/c)^{\$}t}+\frac{(\psi^{\$})^2}{\varphi^{\$}}\left(\frac{1}{\varphi^{\$}}\frac{1}{z^{\$}}-\lambda_3\right)[(\varphi_0-\varphi^{\$})-(\tau_0-\tau^{\$})v_{41}]e^{\lambda_3 t}$$

$$\varphi=\varphi^{\$}+(\tau_0-\tau^{\$})v_{41}e^{-(\dot{c}/c)^{\$}t}+[(\varphi_0-\varphi^{\$})-(\tau_0-\tau^{\$})v_{41}]e^{\lambda_3 t}$$

where v_{21}, v_{31}, v_{41} and λ_3 are constants described in Technical Appendix E.

To get a solution for the first period (t=0), an initial condition for the predetermined variable, φ0, is assumed and then the system is solved backwards. One easily gets: $z_0=z^{\$}+(\tau_0-\tau^{\$})v_{21}$ and $\psi_0=\psi^{\$}+v_{31}(\tau_0-\tau^{\$})+[\varphi_0-\varphi^{\$}-v_{41}(\tau_0-\tau^{\$})]\frac{(\psi^{\$})^2}{\varphi^{\$}}\left(\frac{1}{\varphi^{\$}}\frac{1}{z^{\$}}-\lambda_3\right)$. Note that given $\psi_0>0$, it should be: $\varphi_0>v_{41}(\tau_0-\tau^{\$})-[\psi^{\$}+v_{31}(\tau_0-\tau^{\$})]\frac{\varphi^{\$}}{(\psi^{\$})^2}\left(\frac{1}{\varphi^{\$}}\frac{1}{z^{\$}}-\lambda_3\right)^{-1}+\varphi^{\$}$. Using initial conditions for z and ψ in (11a) and setting t=0 one gets: $\frac{\theta}{2}[1+\frac{1-\alpha}{\alpha}(1-\omega)](1+\alpha z_0\psi_0)$ $(\tau_0-\tau^\$)=(1-\tau_0)(1-\alpha)\frac{1}{k_0}(A\tau_0^{\ 1-\alpha})^{1/\alpha}\{2(1-\omega)(1-\alpha-\tau_0)(1+\alpha z_0\psi_0)+\alpha[(1-\alpha)(2\omega-1)-\tau_0]\}$, from which the initial value for τ (τ_0) may be derived. Note that parameter θ (determining the degree of capital mobility) is present in the above equation. Thus, the degree of capital mobility affects τ0. From the dynamic system above, one may conclude that, through τ0, θ affects also transitional dynamics.

F. Calculation of initial Nash tax rate

Let θ=1, ρ=δ=3%, α=0.8 and k_0=1. Remember that at the Balanced Growth Path it will be: $g^\$=\alpha(1-\tau^\$)\left(\frac{y}{k}\right)^\$-\delta-\rho$, where $\left(\frac{y}{k}\right)^\$=[A(\tau^\$)^{1-\alpha}]^{\frac{1}{\alpha}}$. One may, hence, get A as a function of the above parameter values, $\tau^\$$ and $g^\$$: $A=\frac{1}{\tau^\$}(\frac{g^\$+\delta+\rho}{\alpha}\frac{\tau^\$}{1-\tau^\$})^\alpha$. For the above parameter values and $g^\$$=2.5% one gets: $A=\frac{1}{\tau^\$}(0.10625\frac{\tau^\$}{1-\tau^\$})^{0.8}$. Given that $\tau^\$=(1-\alpha)\frac{\alpha\omega+(1-\alpha)(1-\omega)}{\alpha+(1-\alpha)(1-\omega)}$, A may be specified for specific values of ω. Setting different values for ω one may get the corresponding values of $\tau^\$$ (second column in Table 1).

One may also get specific values for $z^\$$, $\psi^\$$, $\varphi^\$$, v_{21}, $(\dot\psi_\tau)^\$$, $(\dot\psi_z)^\$$, $(\dot\psi_\psi)^\$$, $(\dot\varphi_\tau)^\$$, $(\dot\psi_\psi)^\$$, v_{31}, v_{41} and λ_3. Now the equation system (14a) – (14d) may be written as a function of time, and initial values τ_0 and φ_0.

Setting t=0 in equation (14b) yields to: $z_0=z^\$+v_{21}(\tau_0-\tau^\$)$, i.e. a function relating z_0 to τ_0. One may also get a similar function from equation (14c), relating ψ_0 and τ_0. Using these two functions one may transform equation (11a) into an (non-linear) expression relating τ_0 to φ_0. Thus, for any initial value of φ_0 one may get values for τ_0.

Finally, remember that ψ_0, z_0 and φ_0 should all be positive numbers. Using relationship $\psi_0=\psi^\$+v_{31}(\tau_0-\tau^\$)+[\varphi_0-\varphi^\$-v_{41}(\tau_0-\tau^\$)]\frac{(\psi^\$)^2}{\varphi^\$}(\frac{1}{\varphi^\$}\frac{1}{z^\$}-\lambda_3)$, for any ω and τ_0 value one may find some specific value of φ_0 for which ψ_0=0. Note that ψ_0 depends positively on φ_0. Thus, letting φ_0 be 0.01 higher than the value nullifying the above expression guarantees that $\psi_0>0$. Numerical simulations also confirm that $\varphi_0>0$ and $z_0>0$. For φ_0 values just specified, one may use numerical simulations to finally get τ_0 (3d column in Table 1).

Table 1

	$\tau^{\$}$	τ_0
ω=0.9	0.180	0.174
ω=0.8	0.162	0.153
ω=0.7	0.144	0.137
ω=0.6	0.127	0.123
ω=0.57	0.122	0.12
ω=0.55	0.119	0.117
ω=0.505	0.11190211	0.11190221

where $\theta=1$, $\rho=\delta=3\%$, $\alpha=0.8$, $k_0=1$, $g=2.5\%$ and φ_0 large enough to get $\psi_0>0$

Check, also, that for $\omega=0.5$ it will be: $\tau^{\$}=\frac{1-\alpha}{1+\alpha}$ and eq. (11a) becomes: $\frac{\theta}{2}(1+0.5\frac{1-\alpha}{\alpha})(1+\alpha z_0\psi_0)(\tau_0-\tau^{\$})=(1-\tau_0)(1-\alpha)\frac{1}{k_0}(A\tau_0^{1-\alpha})^{1/\alpha}[(1-\alpha-\tau_0)(1+\alpha z_0\psi_0)-\alpha\tau_0] \Leftrightarrow [(1-\tau_0)(1-\alpha)\frac{1}{k_0}(A\tau_0^{1-\alpha})^{1/\alpha}+\frac{\theta}{2}0.5\frac{1}{\alpha}](1-\alpha-\tau_0)=\tau_0[\alpha(1-\tau_0)(1-\alpha)\frac{1}{k_0}(A\tau_0^{1-\alpha})^{1/\alpha}(1+\alpha z_0\psi_0)^{-1}+\frac{\theta}{2}0.5]>0$, thus $\tau_0<1-\alpha$. It also may be written as: $[(1-\tau_0)(1-\alpha)\frac{1}{k_0}(A\tau_0^{1-\alpha})^{1/\alpha}+\frac{\theta}{2}\frac{1}{\alpha}0.5](1+\alpha)(\tau^{\$}-\tau_0)=-\alpha\tau_0(1-\tau_0)(1-\alpha)\frac{1}{k_0}(A\tau_0^{1-\alpha})^{1/\alpha}\alpha z_0\,\psi_0(1+\alpha z_0\psi_0)^{-1}<0$, thus $\tau_0>\tau^{\$}$.

G. Symmetric Cooperative Solution

Utility maximization results in a current value Hamiltonian of the form:

$$J=\ln(c_j)+\ln(c_f)+v_j[w_j+r_jb_j+\frac{(r_f-r_j)^2}{2\theta}-c_j]+\mu_jc_j(r_j-\rho)+v_f[w_f+r_fb_f+\frac{(r_j-r_f)^2}{2\theta}-c_f]+\mu_fc_f(r_f-\rho).$$

First-order conditions with respect to τ, bj, bf, cj and cf may now be derived.
After some algebra (Technical Appendix F) I prove that in symmetry it should be:

$$\tau=1-\alpha \quad (a)$$

$$\frac{\dot{v}}{v}=\rho+\delta-(1-\tau)\frac{y}{k} \quad (b)$$

$$\frac{\dot{\mu}}{\mu}=\frac{v}{\mu}+\rho-\frac{1}{c}\frac{1}{\mu}-(r-\rho) \quad (c)$$

H. Symmetric Long-run Cooperative Solution

Given that equation (15a) assures the optimal tax rate is constant over time, equation (15b) yields to:

$c = c_0 e^{\left[\alpha(1-\tau)\frac{y}{k}-\delta-\rho\right]t}$, which transforms eq. (15c) to: $\frac{\dot{k}}{k} = (1-\tau)\frac{y}{k} - \delta - \frac{c_0}{k} e^{\left[\alpha(1-\tau)\frac{y}{k}-\delta-\rho\right]t} \Leftrightarrow$

$k = \frac{c_0 e^{\left[\alpha(1-\tau)\frac{y}{k}-\delta-\rho\right]t}}{(1-\alpha)(1-\tau)\frac{y}{k}+\rho} + e^{\left[(1-\tau)\frac{y}{k}-\delta\right]t}\left(k_0 - \frac{c_0}{(1-\alpha)(1-\tau)\frac{y}{k}+\rho}\right).$

Equation (15d) yields to: $v = v_0 e^{\left[\rho+\delta-(1-\tau)\frac{y}{k}\right]t}$. Thus, transversality condition requires:

$\lim_{t\to\infty}(kve^{-\rho t}) = 0 \Leftrightarrow \frac{c_0}{(1-\alpha)(1-\tau)\frac{y}{k}+\rho} v_0 \lim_{t\to\infty}\left\{e^{-\left[(1-\alpha)(1-\tau)\frac{y}{k}+\rho\right]t}\right\} -$

$\left(\frac{c_0}{(1-\alpha)(1-\tau)\frac{y}{k}+\rho} - k_0\right)v_0 = 0 \Leftrightarrow k_0 = \frac{c_0}{(1-\alpha)(1-\tau)\frac{y}{k}+\rho}.$

At the long run equilibrium it should be: $\frac{\dot{k}}{k} = \frac{\dot{c}}{c} \Leftrightarrow \frac{\tilde{c}}{\tilde{k}} = [(1-\alpha)(1-\tilde{\tau})\frac{\tilde{y}}{\tilde{k}} + \rho]$.

The Balanced Growth Path described by $\tilde{\tau} = 1-\alpha$, $\tilde{c} = k_0[(1-\alpha)(1-\tilde{\tau})\frac{\tilde{y}}{\tilde{k}} + \rho]e^{\left[\alpha(1-\tilde{\tau})\frac{\tilde{y}}{\tilde{k}}-\rho-\delta\right]t}$

and $\tilde{k} = k_0 e^{\left[\alpha(1-\tilde{\tau})\frac{\tilde{y}}{\tilde{k}}-\rho-\delta\right]t}$, where $\frac{\tilde{y}}{\tilde{k}} = (A\tilde{\tau}^{1-\alpha})^{\frac{1}{\alpha}}$, will be unique, determined only by pre-determined values (A, α, ρ, δ and k_0). For $\alpha(1-\tilde{\tau})\frac{\tilde{y}}{\tilde{k}} > \delta+\rho \Leftrightarrow \alpha^2[A(1-\alpha)^{1-\alpha}]^{\frac{1}{\alpha}} > \delta+\rho$ it will be $\frac{\dot{k}}{k} = \frac{\dot{c}}{c} > 0$, thus 'real' variables will grow at the same, constant, positive rate.

References

Aghion, P. and Howitt, P., 1998, Endogenous Growth Theory, Cambridge, MA: MIT Press

Alesina, A. and Tabellini, G., 1989, "External Debt, Capital Flight and Political Risk", Journal of International Economics, 44, 199-220.

Alesina, A., and Wacziarg, R., 1999, "Is Europe Going too Far?", Carnegie-Rochester Conference Series on Public Policy, 51, 1-42

Barro, R., and Sala – i – Martin, X., 1995, Economic Growth, McGraw-Hill Inc.

Barro, R., 1990 "Government Spending in a Simple Model of Endogenous Growth", Journal of Political Economy, 98, S103 - S125

Becker, D., and Rauscher, M., 2007, "Fiscal competition in space and time: An endogenous-growth approach", CESifo Working Paper: 2048, Munich.

Blanchard, O.J. and Fischer, S., 1989, Lectures on Macroeconomics, Cambridge, MA: MIT Press

Blanchard, O., and Kahn, Ch., 1980. "The Solution of Linear Difference Models Under Rational Expectations", Econometrica, 48(5), 1305-1311

Bjorvatn, K., and Schjelderup, G., 2002, "Tax competition and international public goods", International Tax and Public Finance, 9, 111-120

Cooper, R. and John, A., 1988, “Coordinating Coordination Failures in Keynesian models”, Quarterly Journal of Economics, 103, 441-63

Epifani, P., and Gancia, G., 2008, “Openness, Government Size and the Terms of Trade”, Institute for Empirical Research in Economics, Working Paper: 359, University of Zurich

Hatfield, J., 2006, “Federalism, taxation, and economic growth”, unpublished paper Graduate School of Business, Stanford University

Kehoe, P., 1987, “Coordination of Fiscal Policies in a World Economy”, Journal of Monetary Economics, 19, 349-76

Koethenbuerger, M., and Lockwood, B., 2007, “Does tax competition really promote growth?”, CESifo Working Paper: 2102, Munich

Lejour, A., and Verbon, H., 1997, “Tax competition and redistribution in a two-country endogenous growth model”, International Tax and Public Finance, 4, 485-497

Mas-Colell A., Winston M. and Green, J., 1995 Microeconomic Theory, Oxford: Oxford University Press

Matsumoto, M., 1998, “A note on tax competition and public input provision”, Regional Science and Urban Economics, 28, 465 – 473

Noiset, L., 1995, “Pigou, Tiebout, property taxation, and the under-provision of local public goods”, Comment Journal of Urban Economics, 38, 312-316

Oates,W.E., 1972 Fiscal Federalism, Harcourt Brace Jovanovich, New York

Persson, T., and Tabellini, G., 1992, “The Politics of 1992: Fiscal Policy and European Integration”, Review of Economics Studies, 59, 689 – 701

Persson, T., and Tabellini, G., 1995, Double – Edged Incentives: Institutions and Policy Coordination, in G. Grossman and K. Rogoff (eds.), Handbook of International Economics, vol. III, Elsevier Science

Persson, T. and Tabellini, G., 1999, Political Economics and Macroeconomic Policy, in J.B. Taylor and M. Woodford, (eds.), Handbook of Macroeconomics, vol. I., Elsevier Science

Philippopoulos, A. and Economides, G., 2003, “Are Nash tax rates too low or too high? The role of endogenous growth in models with public goods”, Review of Economic Dynamics, 6, 37-53

Philippopoulos, A. and Kammas, P., 2007, How harmful is international tax competition?, in G. Korres, (eds.), Regionalization, Growth and Economic Integration, 59-73, Physica-Verlag HD: Springer

Sorensen, P., B., 2000, “The case for international tax co-ordination reconsidered”, Economic Policy, 31, 431-72

Zodrow,G.,R., and Mieszkowski, P., 1986, “Pigou, Tiebout, property taxation and the under-provision of local public goods”, Journal of Urban Economics, 19, 356–370

Technical Appendix

A.

It is: $\frac{\partial r_j}{\partial \tau_j}=\alpha\frac{y_j}{k_j}[(1-\tau_j)(1-\alpha)(\frac{\partial G_j}{\partial \tau_j}\frac{1}{G_j}-\frac{\partial k_j}{\partial \tau_j}\frac{1}{k_j})-1]$, $\frac{\partial w_j}{\partial \tau_j}=(1-\tau_j)(1-\alpha)y_j[(1-\alpha)\frac{\partial G_j}{\partial \tau_j}\frac{1}{G_j}+\frac{1}{k_j}\alpha\frac{\partial k_j}{\partial \tau_j}]-(1-\alpha)y_j$ and $\frac{\partial w_j}{\partial \tau_j}+k_j\frac{\partial r_j}{\partial \tau_j}=y_j[(1-\tau_j)(1-\alpha)\frac{\partial G_j}{\partial \tau_j}\frac{1}{G_j}-1]$.

It will be: $k_j=b_{j,j}+b_{f,j}=b_{j,j}-\frac{r_f-r_j}{\theta}$. Also it is $b_j=b_{j,j}+b_{j,f}\Leftrightarrow b_{j,j}=b_j-\frac{r_f-r_j}{\theta}$, thus I finally get: $k_j=b_j+\frac{2}{\theta}(r_j-r_f)$. Similarly I get: $k_f=b_f-\frac{2}{\theta}(r_j-r_f)$. It will, thus, be: $\frac{\partial k_j}{\partial \tau_j}=\frac{2}{\theta}\frac{\partial(r_j-r_f)}{\partial \tau_j}$ and $\frac{\partial k_f}{\partial \tau_j}=\frac{2}{\theta}\frac{\partial(r_f-r_j)}{\partial \tau_j}=-\frac{\partial k_j}{\partial \tau_j}$.

Using equation (3b) in (3a) one gets: $G_j^{\alpha[1+(1-\alpha)(1-\omega j-\omega f)]}=(\tau_f A_f k_f^{\alpha})^{(1-\omega j)}(\tau_j A_j k_j^{\alpha})^{\omega j+(1-\alpha)(1-\omega j-\omega f)}$. It will, hence, be: $[1+(1-\alpha)(1-\omega_j-\omega_f)]\frac{1}{G_j}\frac{\partial G_j}{\partial \tau_j}=[\omega_j+(1-\alpha)(1-\omega_j-\omega_f)]\frac{1}{\tau_j}\frac{1}{\alpha}-(1-\omega_j)\frac{\partial k_j}{\partial \tau_j}(\frac{1}{k_f}+\frac{1}{k_j})+[1+(1-\alpha)(1-\omega_j-\omega_f)]\frac{1}{k_j}\frac{\partial k_j}{\partial \tau_j}$, while for economy f: $[1+(1-\alpha)(1-\omega_j-\omega_f)]\frac{1}{G_f}\frac{\partial G_f}{\partial \tau_j}=(1-\omega_f)\frac{1}{\tau_j}\frac{1}{\alpha}+(1-\omega_f)\frac{\partial k_j}{\partial \tau_j}(\frac{1}{k_f}+\frac{1}{k_j})-[1+(1-\alpha)(1-\omega_j-\omega_f)]\frac{1}{k_f}\frac{\partial k_j}{\partial \tau_j}$.

Thus it will be: $\frac{\partial k_j}{\partial \tau_j}=\frac{2}{\theta}\frac{\partial(r_j-r_f)}{\partial \tau_j}=\frac{2}{\theta}[\alpha(1-\tau_j)\frac{y_j}{k_j}(1-\alpha)(\frac{1}{G_j}\frac{\partial G_j}{\partial \tau_j}-\frac{1}{k_j}\frac{\partial k_j}{\partial \tau_j})-\alpha\frac{y_j}{k_j}-\alpha(1-\alpha)(1-\tau_f)\frac{y_f}{k_f}(\frac{1}{G_f}\frac{\partial G_f}{\partial \tau_j}+\frac{1}{k_f}\frac{\partial k_j}{\partial \tau_j})]\Leftrightarrow\{\frac{\theta}{2}[1+(1-\alpha)(1-\omega_j-\omega_f)]+\alpha(1-\tau_j)\frac{y_j}{k_j}(1-\alpha)(1-\omega_j)(\frac{1}{k_f}+\frac{1}{k_j})+\alpha(1-\alpha)(1-\tau_f)\frac{y_f}{k_f}(1-\omega_f)(\frac{1}{k_f}+\frac{1}{k_j})\}\frac{\partial k_j}{\partial \tau_j}=\frac{y_j}{k_j}\{(1-\tau_j)(1-\alpha)[\omega_j+(1-\alpha)(1-\omega_j-\omega_f)]\frac{1}{\tau_j}-\alpha[1+(1-\alpha)(1-\omega_j-\omega_f)]\}-\frac{y_f}{k_f}(1-\alpha)(1-\tau_f)(1-\omega_f)$, which in symmetry becomes: $\frac{\partial k_j}{\partial \tau_j}=\frac{y}{k}\alpha\frac{1}{\tau}[(1-\alpha)(2\omega-1)-\tau]\{\frac{\theta}{2}[1-(1-\alpha)(2\omega-1)]+4\alpha(1-\tau)(1-\alpha)(1-\omega)\frac{y}{k}\frac{1}{k}\}^{-1}$.

Now $\frac{\partial G_j}{\partial \tau_j}\frac{1}{G_J}$ can be determined and through it, one gets $\frac{\partial r_f}{\partial \tau_j}$, $\frac{\partial r_j}{\partial \tau_j}$, $\frac{\partial w_j}{\partial \tau_j}$.

Note that, since $b_j=k_j-\frac{2}{\theta}(r_j-r_f)$, it will be: $\frac{\partial w_j}{\partial \tau_j}+b_j\frac{\partial r_j}{\partial \tau_j}+\frac{r_f-r_j}{\theta}(\frac{\partial r_f}{\partial \tau_j}-\frac{\partial r_j}{\partial \tau_j})=\frac{\partial w_j}{\partial \tau_j}+\frac{\partial r_j}{\partial \tau_j}k_j+\frac{r_f-r_j}{\theta}(\frac{\partial r_j}{\partial \tau_j}+\frac{\partial r_f}{\partial \tau_j})$, converting the First Order Condition to: $\mu_j c_j\frac{\partial r_j}{\partial \tau_j}+v_j[\frac{\partial w_j}{\partial \tau_j}+\frac{\partial r_j}{\partial \tau_j}k_j+\frac{r_f-r_j}{\theta}(\frac{\partial r_j}{\partial \tau_j}+\frac{\partial r_f}{\partial \tau_j})]=0$.

In symmetry, it will be $r_f=r_j$ and the First Order Condition will finally be given by:

$$\mu c\frac{\partial r_j}{\partial \tau_j}+v(\frac{\partial w_j}{\partial \tau_j}+\frac{\partial r_j}{\partial \tau_j}k)=0 \quad \Leftrightarrow \quad \mu c\alpha\frac{y}{k}[(1-\tau)(1-\alpha)(\frac{\partial G_j}{\partial \tau_j}\frac{1}{G_j}-\frac{\partial k_j}{\partial \tau_j}\frac{1}{k_j})-1]+vy[(1-\tau)(1-\alpha)\frac{\partial G_j}{\partial \tau_j}\frac{1}{G_j}-1]=0.$$

B.

It will be: $\frac{\partial k_j}{\partial b_j}=1+\frac{2}{\theta}(\frac{\partial r_j}{\partial b_j}-\frac{\partial r_f}{\partial b_j})$, $\frac{\partial k_f}{\partial b_j}=\frac{2}{\theta}(\frac{\partial r_f}{\partial b_j}-\frac{\partial r_j}{\partial b_j})=1-\frac{\partial k_j}{\partial b_j}$ and $\frac{\partial r_j}{\partial b_j}=\alpha(1-\alpha)(1-\tau_j)\frac{y_j}{k_j}(\frac{\partial G_j}{\partial b_j}\frac{1}{G_j}-\frac{1}{k_j}\frac{\partial k_j}{\partial b_j})$ and $\frac{\partial r_f}{\partial b_j}=\alpha(1-\alpha)(1-\tau_f)\frac{y_f}{k_f}(\frac{\partial G_f}{\partial b_f}\frac{1}{G_f}-\frac{1}{k_f}\frac{\partial k_f}{\partial b_j})$. After some Algebra one gets: $\frac{\partial k_j}{\partial b_j}\{\frac{\theta}{2}[1+(1-\alpha)(1-\omega_j-\omega_f)]+\alpha(1-\alpha)(\frac{1}{k_j}+\frac{1}{k_f})[(1-\tau_j)\frac{y_j}{k_j}(1-\omega_j)+(1-\tau_f)\frac{y_f}{k_f}(1-\omega_f)]\}=\frac{\theta}{2}[1+(1-\alpha)(1-\omega_j-\omega_f)]+\alpha(1-\alpha)\frac{1}{k_f}[(1-\tau_j)\frac{y_j}{k_j}(1-\omega_j)+(1-\tau_f)\frac{y_f}{k_f}(1-\omega_f)]$.

In symmetry this will be: $\frac{\partial k_j}{\partial b_j}=\frac{\theta}{2}\{[1+(1-\alpha)(1-2\omega)]+2\alpha(1-\alpha)(1-\tau)(1-\omega)\}\frac{y}{k}\frac{1}{k}\{\frac{\theta}{2}[1+(1-\alpha)(1-2\omega)]+4\alpha(1-\alpha)(1-\tau)(1-\omega)\frac{y}{k}\frac{1}{k}\}^{-1}$ and $\frac{\partial G_j}{\partial b_j}\frac{1}{G}=\frac{1}{k}\{\frac{\theta}{2}[\alpha\omega+(1-\alpha)(1-\omega)]+2\alpha(1-\omega)(1-\alpha)(1-\tau)\frac{y}{k}\frac{1}{k}\}\{\frac{\theta}{2}[1+(1-\alpha)(1-2\omega)]+4\alpha(1-\alpha)(1-\tau)(1-\omega)\frac{y}{k}\frac{1}{k}\}^{-1}$.

Thus, one gets: $\frac{\partial w_j}{\partial \tau_j}+k\frac{\partial r_j}{\partial b_j}=(1-\alpha)(1-\tau)\frac{y}{k}\{\frac{\theta}{2}[\alpha\omega+(1-\alpha)(1-\omega)]+2\alpha(1-\alpha)(1-\tau)(1-\omega)\frac{y}{k}\frac{1}{k}\}\{\frac{\theta}{2}[1-(1-\alpha)(2\omega-1)]+4\alpha(1-\alpha)(1-\tau)(1-\omega)\frac{y}{k}\frac{1}{k}\}^{-1}$ and the second FOC becomes: $\frac{\dot{v}}{v}=\rho-r-\frac{y}{k}(1-\alpha)(1-\tau)\{\frac{\theta}{2}[\alpha\omega+(1-\alpha)(1-\omega)]+2\alpha(1-\alpha)(1-\tau)(1-\omega)\frac{y}{k}\frac{1}{k}-\alpha\frac{\mu}{v}\frac{c}{k}(1-\omega)\frac{\theta}{2}\}\{\frac{\theta}{2}[1-(1-\alpha)(2\omega-1)]+4\alpha(1-\alpha)(1-\tau)(1-\omega)\frac{y}{k}\frac{1}{k}\}^{-1}$.

C.

Eq. (11a) may be written as: $\frac{\theta}{2}[1+\frac{1-\alpha}{\alpha}(1-\omega)](1+\alpha\psi z)[(1-\alpha)\frac{\alpha\omega+(1-\alpha)(1-\omega)}{\alpha+(1-\alpha)(1-\omega)}-\tau]+(1-\tau)(1-\alpha)\frac{1}{k}\frac{y}{k}\{2(1-\omega)(1-\alpha-\tau)(1+\alpha\psi z)+\alpha[(1-\alpha)(2\omega-1)-\tau]\}=0\Leftrightarrow\alpha\psi z\{\frac{\theta}{2}[1+\frac{1-\alpha}{\alpha}(1-\omega)][\tau-(1-\alpha)\frac{\alpha\omega+(1-\alpha)(1-\omega)}{\alpha+(1-\alpha)1-\omega}]-(1-\tau)(1-\alpha)\frac{1}{k}\frac{y}{k}2(1-\omega)(1-\alpha-\tau)\}=(1-\tau)(1-\alpha)\frac{1}{k}\frac{y}{k}$

$\{(1-\alpha)[2(1-\omega)+(2\omega-1)\alpha]-[2(1-\omega)+\alpha]\tau\}-\frac{\theta}{2}[1+\frac{1-\alpha}{\alpha}(1-\omega)][\tau-\frac{\alpha\omega+(1-\alpha)(1-\omega)}{\alpha+(1-\alpha)1-\omega}(1-\alpha)]$.

Let $RHS\equiv\{(1-\tau)(1-\alpha)\frac{1}{k}\frac{y}{k}[\frac{2(1-\omega)+\alpha(2\omega-1)}{2(1-\omega)+\alpha}(1-\alpha)-\tau][2(1-\omega)+\alpha]-\frac{\theta}{2}[1+\frac{1-\alpha}{\alpha}(1-\omega)][\tau-(1-\alpha)\frac{\alpha\omega+(1-\alpha)(1-\omega)}{\alpha+(1-\alpha)1-\omega}]\}\{[\tau-\frac{\alpha\omega+(1-\alpha)(1-\omega)}{\alpha+(1-\alpha)1-\omega}(1-\alpha)][1+\frac{1-\alpha}{\alpha}(1-\omega)]-(1-\tau)(1-\alpha)2\frac{1}{k}\frac{y}{k}(1-\omega)(1-\alpha-\tau)\}^{-1}$.

Remember that in symmetry, ${}^{1-\alpha}/_{\alpha}$ at any point of time, it will be: $\frac{y}{k}=A^{1/\alpha}\tau$. Now let: $s_1\equiv(1-\alpha)[2(1-\omega)+\alpha]A^{1/\alpha}$, $s_2\equiv\frac{\theta}{2}[1+\frac{1-\alpha}{\alpha}(1-\omega)]$, $s_3\equiv2(1-\alpha)(1-\omega)A^{1/\alpha}$, $s_4\equiv[\alpha+2(1-\omega)(1-\alpha)]\frac{1-\alpha}{\alpha+2(1-\omega)}$, $\tilde{\tau}=1-\alpha$ and $\tau^{@}\equiv(1-\alpha)\frac{\alpha\omega+(1-\alpha)(1-\omega)}{\alpha+(1-\alpha)1-\omega}$. It will be: $RHS=[s_1(1-\tau)\frac{1}{k}\tau^{1-\alpha/\alpha}(s_4-\tau)-s_2(\tau-\tau^{@})][s_2(\tau-\tau^{@})-s_3(1-\tau)\frac{1}{k}\tau^{1-\alpha/\alpha}(\tilde{\tau}-\tau)]^{-1}$ and $\frac{1}{RHS}\frac{\partial RHS}{\partial t}=-[s_2(\tau^{@}-\tau)+(1-\tau)\frac{1}{k}\tau^{1-\alpha/\alpha}s_1(s_4-\tau)]^{-1}\{\dot{\tau}[\frac{1}{k}\tau^{1-\alpha/\alpha}s_1(s_4-\tau)+s_1(1-\tau)\frac{1}{k}\tau^{1-\alpha/\alpha}+s_2+\frac{1-\alpha}{\alpha}\frac{1-\tau}{\tau}\frac{1}{k}\tau^{1-\alpha/\alpha}s_1(\tau-s_4)]+(1-\tau)\frac{\dot{k}}{k}\frac{1}{k}\tau^{1-\alpha/\alpha}s_1(s_4-\tau)\}-[s_2(\tau^{@}-\tau)+\frac{1}{k}s_3\tau^{1-\alpha/\alpha}(1-\tau)(\tilde{\tau}-\tau)]^{-1}\{\dot{\tau}[\frac{1}{k}\tau^{1-\alpha/\alpha}s_3(\tau-\tilde{\tau})-(1-\tau)\frac{1}{k}\tau^{1-\alpha/\alpha}s_3-s_2-\frac{1-\tau}{\tau}\frac{1}{k}\tau^{1-\alpha/\alpha}s_3(\tau-\tilde{\tau})\frac{1-\alpha}{\alpha}]+(1-\tau)\frac{1}{k}\tau^{1-\alpha/\alpha}s_3(\tau-\tilde{\tau})\frac{\dot{k}}{k}\}$ $\Leftrightarrow$ $\frac{1}{RHS}\frac{\partial RHS}{\partial t}[s_2(\tau^{@}-\tau)+(1-\tau)\frac{1}{k}\tau^{1-\alpha/\alpha}s_1(s_4-\tau)][s_2(\tau^{@}-\tau)+(1-\tau)\frac{1}{k}\tau^{1-\alpha/\alpha}s_3(\tilde{\tau}-\tau)]+(1-\tau)\frac{\dot{k}}{k}\frac{1}{k}\tau^{1-\alpha/\alpha}(\tau-\tau^{@})\quad s_2[s_1(\tau-s_4)+s_3(\tilde{\tau}-\tau)]=\dot{\tau}\frac{1}{k}\tau^{1-\alpha/\alpha}s_2\{[s_3(\tau-\tilde{\tau})+s_1(s_4-\tau)][1-\tau^{@}-(\tau-\tau^{@})\frac{1-\alpha}{\alpha}\frac{1-\tau}{\tau}]-(1-\tau)(\tau-\tau^{@})(s_3-s_1)\}+\dot{\tau}[(1-\tau)\frac{1}{k}\tau^{1-\alpha/\alpha}]^2s_3s_1(s_4-\tilde{\tau})$.

From equation (11a) I get: $(1-\tau)\frac{1}{k}\tau^{1-\alpha/\alpha}=s_2(1+\alpha\psi z)(\tau-\tau^{@})[s_3\alpha\psi z(\tilde{\tau}-\tau)+s_1(s_4-\tau)]^{-1}$.

Let $\xi\equiv s_2(1+\alpha\psi z)(\tau-\tau^{@})[s_3\alpha\psi z(\tilde{\tau}-\tau)+s_1(s_4-\tau)]^{-1}$. Now it will be: $\frac{\dot{\tau}}{\tau}=\frac{1-\tau}{\tau}\{\frac{\partial RHS/\partial t}{RHS}\frac{1}{\xi}[s_2(\tau^{@}-\tau)+\xi s_3(\tilde{\tau}-\tau)][\xi s_1(s_4-\tau)-s_2(\tau-\tau^{@})]+\frac{\dot{k}}{k}s_2(\tau^{@}-\tau)[s_1(s_4-\tau)+s_3(\tau-\tilde{\tau})]\}\{s_2\{[s_3(\tau-\tilde{\tau})+s_1(s_4-\tau)][1-\tau^{@}-(\tau-\tau^{@})\frac{1-\alpha}{\alpha}\frac{1-\tau}{\tau}]-(1-\tau)(\tau-\tau^{@})(s_3-s_1)\}+s_1s_3\xi(1-\tau)(s_4-\tilde{\tau})\}^{-1}$.

Now, from eq. (11e) and (11d) one gets: $\frac{\dot{\psi}}{\psi}=\frac{\dot{\mu}}{\mu}-\frac{\dot{v}}{v}=\frac{1}{\psi}+\rho-\frac{1}{z}\frac{1}{\varphi}+(1-\alpha)(1-\tau)A^{1/\alpha}$

$\tau^{1-\alpha/\alpha}\{\frac{\theta}{2}[\alpha\omega+(1-\alpha)(1-\omega)]+2\alpha(1-\alpha)(1-\tau)(1-\omega)A^{1/\alpha}\tau^{1-\alpha/\alpha}\frac{1}{k}-\alpha\psi z(1-\omega)\frac{\theta}{2}\}\{\frac{\theta}{2}[1-(1-\alpha)(2\omega-1)]+4\alpha(1-\alpha)A^{1/\alpha}\tau^{1-\alpha/\alpha}(1-\omega)(1-\tau)\frac{1}{k}\}^{-1}$. It is: $\frac{\theta}{2}[1-(1-\alpha)(2\omega-1)]+4\alpha(1-\alpha)A^{1/\alpha}\tau^{1-\alpha/\alpha}(1-\omega)(1-\tau)\frac{1}{k}=\frac{\theta}{2}\{2(1-\omega)(1-\alpha-\tau)(1+\alpha\psi z)+\alpha[(1-\alpha)(2\omega-1)-\tau]\}^{-1}[2(1-\omega)(1+\alpha\psi z)-1-(1-\alpha)(1-2\omega)]\alpha[\tau-(1-\alpha)(2\omega-1)]$ and $\frac{\theta}{2}[\alpha\omega+(1-\alpha)(1-\omega)]+2\alpha(1-\alpha)(1-\tau)(1-\omega)A^{1/\alpha}\tau^{1-\alpha/\alpha}\frac{1}{k}-\alpha\psi z(1-\omega)\frac{\theta}{2}=\frac{\theta}{2}[2(1-\omega)(1+\alpha\psi z)-1-(1-\alpha)(1-2\omega)]\{\tau[\alpha+(1-\omega)(1+\alpha\psi z)]-(1-\alpha)[\alpha(2\omega-1)+(1-\omega)(1+\alpha\psi z)]\}\{2(1-\omega)(1-\alpha-\tau)(1+\alpha\psi z)+\alpha[(1-\alpha)(2\omega-1)-\tau]\}^{-1}$, thus one gets:

$\frac{\dot{\psi}}{\psi}=\frac{1}{\psi}+\rho-\frac{1}{z}\frac{1}{\varphi}+(A\tau)^{1/\alpha}\frac{1-\tau}{\tau}\frac{1-\alpha}{\alpha}\{\tau[\alpha+(1-\omega)(1+\alpha\psi z)]-(1-\alpha)[\alpha(2\omega-1)+(1-\omega)(1+\alpha\psi z)]\}[\tau-(2\omega-1)(1-\alpha)]^{-1}$.

Equation (11a) requires: RHS=$\alpha\psi z$, thus $\frac{1}{RHS}\frac{\partial RHS}{\partial t}=\frac{\dot{z}}{z}+\frac{\dot{\psi}}{\psi}$.

Now it will be: $\frac{\dot{\tau}}{\tau}=\frac{1-\tau}{\tau}\{[z+\frac{1}{\psi}-\frac{1}{z}\frac{1}{\varphi}-\frac{1}{\alpha}\frac{1}{\tau-(2\omega-1)(1-\alpha)}(1-\alpha)(1-\tau)(1-\omega)(1+\alpha\psi z)(1-\alpha-\tau)(A\tau^{1-\alpha})^{1/\alpha}]\frac{1}{\xi}[s_2(\tau^{@}-\tau)+\xi s_3(\tilde{\tau}-\tau)][\xi s_1(s_4-\tau)-s_2(\tau-\tau^{@})]+[(1-\tau)(A\tau^{1-\alpha})^{1/\alpha}-\delta-z]s_2(\tau^{@}-\tau)[s_1(s_4-\tau)+s_3(\tau-\tilde{\tau})]\}\{s_2\{[s_3(\tau-\tilde{\tau})+s_1(s_4-\tau)][1-\tau^{@}-(\tau-\tau^{@})\frac{1-\alpha}{\alpha}\frac{1-\tau}{\tau}]-(1-\tau)(\tau-\tau^{@})(s_3-s_1)\}+s_1s_3\,\xi(1-\tau)(s_4-\tilde{\tau})\}^{-1}$.

D.

Equation (12a) will be at the long run equilibrium:

$\frac{1-\tau^{\$}}{\tau^{\$}}\{[z^{\$}+\frac{1}{\psi^{\$}}-\frac{1}{z^{\$}}\frac{1}{\varphi^{\$}}-\frac{1}{\alpha}\frac{1}{\tau^{\$}-(2\omega-1)(1-\alpha)}(1-\alpha)(1-\tau^{\$})(1-\omega)(1+\alpha\psi^{\$}z^{\$})(1-\alpha-\tau^{\$})A^{1/\alpha}(\tau^{\$})^{1-\alpha/\alpha}]\frac{1}{\xi^{\$}}[s_2(\tau^{@}-\tau^{\$})+\xi^{\$}s_3(\tilde{\tau}-\tau^{\$})][\xi^{\$}s_1(s_4-\tau^{\$})-s_2(\tau^{\$}-\tau^{@})]+[(1-\tau^{\$})A^{1/\alpha}(\tau^{\$})^{1-\alpha/\alpha}-\delta-z^{\$}]s_2(\tau^{@}-\tau^{\$})[s_1(s_4-\tau^{\$})+s_3(\tau^{\$}-\tilde{\tau})]\}\{s_2\{[s_3(\tau^{\$}-\tilde{\tau})+s_1(s_4-\tau^{\$})][1-\tau^{@}-(\tau^{\$}-\tau^{@})\frac{1-\alpha}{\alpha}\frac{1-\tau^{\$}}{\tau^{\$}}]-(1-\tau^{\$})(\tau^{\$}-\tau^{@})(s_3-s_1)\}+s_1s_3\xi^{\$}(1-\tau^{\$})(s_4-\tilde{\tau})\}^{-1}=0$, which for $s_2\{[s_3(\tau^{\$}-\tilde{\tau})+s_1(s_4-\tau^{\$})][1-\tau^{@}-(\tau^{\$}-\tau^{@})$

$\frac{1-\alpha}{\alpha}\frac{1-\tau^{\$}}{\tau^{\$}}$]-(1-$\tau^{\$}$)($\tau^{\$}$-$\tau^{@}$)($s_3$-$s_1$)}+$s_1 s_3 \xi^{\$}$(1-$\tau^{\$}$)($s_4$-$\tilde{\tau}$)≠0 and ξ≠0, becomes: $s_2(\tau^{\$}-\tau^{@})[\alpha(1-\tau^{\$})A^{1/\alpha}(\tau^{\$})^{1-\alpha/\alpha}-\delta-\rho]\ [s_1(s_4-\tau^{\$})+s_3(\tau^{\$}-\tilde{\tau})]=0$. One solution may, hence, be: $\tau^{\$}=\tau^{@}\equiv(1-\alpha)\frac{\alpha\omega+(1-\alpha)(1-\omega)}{\alpha+(1-\alpha)1-\omega}$. Solution $\alpha(1-\tau^{\$})A^{1/\alpha}(\tau^{\$})^{1-\alpha/\alpha}=\delta+\rho$ is not acceptable, because in that case the growth rate of consumption will be zero. Finally, it may be: $s_1(s_4-\tau^{\$})+s_3(\tau^{\$}-\tilde{\tau})=0 \Leftrightarrow \tau^{\$}=\frac{1-\alpha}{\alpha}[\alpha+2(1-\omega)(1-\alpha)-2(1-\omega)]=(1-\alpha)(2\omega-1)>0$. Note, however, that in that case it will also be: $(1-\omega)(1+\alpha\psi^{\$}z^{\$})(1-\alpha-\tau^{\$})=\alpha[\tau^{\$}-(2\omega-1)(1-\alpha)]=0$ $\Leftrightarrow \psi^{\$}z^{\$}<0$, which is not acceptable. Thus, $\tau^{\$}=(1-\alpha)\frac{\alpha\omega+(1-\alpha)(1-\omega)}{\alpha+(1-\alpha)1-\omega}$, will be the only valid solution.

In case ξ=0, given that $(1-\omega)(1+\alpha\psi^{\$}z^{\$})(1-\alpha-\tau^{\$})=\alpha[\tau^{\$}-(2\omega-1)(1-\alpha)]$, it will be: $[\tau^{\$}-(2\omega-1)(1-\alpha)](\tau^{\$}-\tau^{@})(1-\alpha-\tau^{\$})^{-1}\{(1-\omega)^{-1}(1-\alpha-\tau^{\$})^{-1}\alpha[\tau^{\$}-(2\omega-1)(1-\alpha)]s_3(\tilde{\tau}-\tau^{\$})-s_3(\tilde{\tau}-\tau^{\$})+s_1(s_4-\tau^{\$})\}^{-1}=0$ and since $\tau^{\$}\neq(2\omega-1)(1-\alpha)$ this again yields to: $(\tau^{\$}-\tau^{@})(1-\alpha-\tau^{\$})^{-1}=0$ $\Leftrightarrow \tau^{\$}=\tau^{@}$.

Finally, for $\tau^{\$}=\tau^{@}$ it will be: $s_2\{[s_3(\tau^{\$}-\tilde{\tau})+s_1(s_4-\tau^{\$})][1-\tau^{@}-(\tau^{\$}-\tau^{@})\frac{1-\alpha}{\alpha}\frac{1-\tau^{\$}}{\tau^{\$}}]-(1-\tau^{\$})(\tau^{\$}-\tau^{@})(s_3-s_1)\}+s_1 s_3\xi^{\$}(1-\tau^{\$})(s_4-\tilde{\tau})=s_2[s_3(\tau^{\$}-\tilde{\tau})+s_1(s_4-\tau^{\$})](1-\tau^{@})=-\frac{\theta}{2}(1-\alpha)^2A^{1/\alpha}(1-\omega)[\alpha+2(1-\alpha)(1-\omega)](1-\tau^{@})\neq 0$.

E.

Linearizing the system described by equations (12a)-(12d) around the Balanced Growth Path described by equations (13a)-(13d), produces the following Jacobian matrix:

$$J=\begin{bmatrix} -\left(\frac{\dot{c}}{c}\right)^{\$} & 0 & 0 & 0 \\ -z^{\$}\frac{(1-\alpha)(1-\omega)}{\alpha\omega+(1-\alpha)(1-\omega)}\left(A\tau^{\$^{1-\alpha}}\right)^{1/\alpha} & z^{\$} & 0 & 0 \\ (\dot{\psi}_{\tau})^{\$} & (\dot{\psi}_{z})^{\$} & (\dot{\psi}_{\psi})^{\$} & \frac{\psi^{\$}}{(\varphi^{\$})^2}\frac{1}{z^{\$}} \\ (\dot{\varphi}_{\tau})^{\$} & (\dot{\varphi}_{z})^{\$} & -\frac{\varphi^{\$}}{(\psi^{\$})^2} & \frac{1}{\varphi^{\$}}\frac{1}{z^{\$}} \end{bmatrix}$$

, where it

will be:

$(\dot{\psi}_{\tau})^{\$}=2\psi^{\$}[A(\tau^{\$})^{1-\alpha}]^{1/\alpha}(\frac{\alpha}{1-\omega}+1-\alpha)>0$, $(\dot{\psi}_{z})^{\$}=\frac{\psi^{\$}}{z^{\$}}\{\frac{\alpha(1-\omega)z^{\$}}{\alpha\omega+(1-\alpha)(1-\omega)}+\rho-[A(\tau^{\$})^{1-\alpha}]^{1/\alpha}\alpha\tau^{\$}\}$,

$(\dot{\psi}_{\psi})^{\$}=\rho-\frac{1}{o^{\$}}\frac{1}{z^{\$}}-(z^{\$}-\rho)\frac{1-\omega+\alpha(2\omega-1)}{\alpha+2(1-\omega)(1-\alpha)}=-\frac{\alpha(1-\omega)}{\alpha\omega+(1-\alpha)(1-\omega)}z^{\$}-(1-\alpha)\frac{(1-\omega)+\alpha(2\omega-1)}{\alpha+2(1-\omega)(1-\alpha)}(1-\tau^{\$})[A(\tau^{\$})^{1-\alpha}]^{1/\alpha}<0$,

$(\dot{\varphi}_{\tau})^{\$}=\varphi^{\$}[A(\tau^{\$})^{1-\alpha}]^{1/\alpha}\frac{(1-\alpha)(1-\omega)}{\alpha\omega+(1-\alpha)(1-\omega)}$ and $(\dot{\varphi}_{z})^{\$}=\frac{1}{(z^{\$})^{2}}-\varphi^{\$}$.

It will be:

$|J|=-\left(\frac{\dot{c}}{c}\right)^{\$}z^{\$}[(\dot{\psi}_{\psi})^{\$}\frac{1}{\varphi^{\$}}\frac{1}{z^{\$}}+\frac{\varphi^{\$}}{(\psi^{\$})^{2}}\frac{\psi^{\$}}{(\varphi^{\$})^{2}}\frac{1}{z^{\$}}]=-\left(\frac{\dot{c}}{c}\right)^{\$}\frac{1}{\varphi^{\$}}[(\dot{\psi}_{\psi})^{\$}+\frac{1}{\psi^{\$}}]$. Remember that: $\frac{1}{\psi^{\$}}=\frac{1}{\varphi^{\$}}\frac{1}{z^{\$}}-\rho$, thus: $(\dot{\psi}_{\psi})^{\$}+\frac{1}{\psi^{\$}}=-\frac{(1-\omega)+\alpha(2\omega-1)}{\alpha+2(1-\omega)(1-\alpha)}(z^{\$}-\rho)<0$, yielding to: $|J|>0$, thus there will be either no, two or four positive eigenvalues.

It is: $|J-\lambda I|=0 \Leftrightarrow$

$$\begin{vmatrix} -\left(\frac{\dot{c}}{c}\right)^{\$}-\lambda & 0 & 0 & 0 \\ -z^{\$}\frac{(1-\alpha)(1-\omega)}{\alpha\omega+(1-\alpha)(1-\omega)}(A\tau^{\$1-\alpha})^{1/\alpha} & z^{\$}-\lambda & 0 & 0 \\ (\dot{\psi}_{\tau})^{\$} & (\dot{\psi}_{z})^{\$} & (\dot{\psi}_{\psi})^{\$}-\lambda & \frac{\psi^{\$}}{(\varphi^{\$})^{2}}\frac{1}{z^{\$}} \\ (\dot{\varphi}_{\tau})^{\$} & (\dot{\varphi}_{z})^{\$} & -\frac{\varphi^{\$}}{(\psi^{\$})^{2}} & \frac{1}{\varphi^{\$}}\frac{1}{z^{\$}}-\lambda \end{vmatrix}$$

$=0 \Leftrightarrow (-\frac{\dot{c}}{c}-\lambda)(z^{\$}-\lambda)\{[(\dot{\psi}_{\psi})^{\$}-\lambda](\frac{1}{\varphi^{\$}}\frac{1}{z^{\$}}-\lambda)+\frac{1}{\varphi^{\$}}\frac{1}{z^{\$}}\frac{1}{\psi^{\$}}\}=0$, $\lambda_1=-\left(\frac{\dot{c}}{c}\right)^{\$}<0$, $\lambda_2=z^{\$}>0$ while λ_3 and λ_4 will be the roots of: $\lambda^2-\lambda[(\dot{\psi}_{\psi})^{\$}+\frac{1}{\varphi^{\$}}\frac{1}{z^{\$}}]+\frac{1}{\varphi^{\$}}\frac{1}{z^{\$}}[(\dot{\psi}_{\psi})^{\$}+\frac{1}{\psi^{\$}}]=0 \Leftrightarrow \lambda^2-\lambda[\rho-(z^{\$}-\rho)\frac{(1-\omega)+\alpha(2\omega-1)}{\alpha+2(1-\omega)(1-\alpha)}]-(z^{\$}-\rho)[\frac{\alpha(1-\omega)}{\alpha\omega+(1-\alpha)(1-\omega)}z^{\$}+\rho]\frac{(1-\omega)+\alpha(2\omega-1)}{\alpha+2(1-\omega)(1-\alpha)}=0$. It will be: $\Delta=[\rho-(z^{\$}-\rho)\frac{(1-\omega)+\alpha(2\omega-1)}{\alpha+2(1-\omega)(1-\alpha)}]^{2}+4\frac{(1-\omega)+\alpha(2\omega-1)}{\alpha+2(1-\omega)(1-\alpha)}(z^{\$}-\rho)[$

$\frac{\alpha(1-\omega)}{\alpha\omega+(1-\alpha)(1-\omega)}z^{\$}+\rho]>0$, thus $\lambda_3=0.5[\rho-(z^{\$}-\rho)\frac{(1-\omega)+\alpha(2\omega-1)}{\alpha+2(1-\omega)(1-\alpha)}-\sqrt{\Delta}]$ and $\lambda_4=0.5[\rho-(z^{\$}-\rho)\frac{(1-\omega)+\alpha(2\omega-1)}{\alpha+2(1-\omega)(1-\alpha)}+\sqrt{\Delta}]$.

There will, hence, be two positive and two negative eigenvalues.

The related eigenvectors to λ_1, will be: $v_{11}=1$, $v_{21}=z^{\$}[A(\tau^{\$})^{1-\alpha}]^{1/\alpha}\frac{(1-\omega)(1-\alpha)}{\alpha\omega+(1-\omega)(1-\alpha)}[z^{\$}+\left(\frac{\dot{c}}{c}\right)^{\$}]^{-1}$, $v_{31}=-[\frac{\psi^{\$}}{(\varphi^{\$})^2}\frac{1}{z^{\$}}v_{41}+(\dot{\psi}_{\tau})^{\$}+(\dot{\psi}_{z})^{\$}v_{21}][(\dot{\psi}_{\psi})^{\$}+\left(\frac{\dot{c}}{c}\right)^{\$}]^{-1}$ and $v_{41}=-\{[(\dot{\psi}_{\tau})^{\$}+(\dot{\psi}_{z})^{\$}v_{21}]\frac{\varphi^{\$}}{(\psi^{\$})^2}+[(\dot{\psi}_{\psi})^{\$}+\left(\frac{\dot{c}}{c}\right)^{\$}][(\dot{\varphi}_{\tau})^{\$}+(\dot{\varphi}_{z})^{\$}v_{21}]\}\{[(\dot{\psi}_{\psi})^{\$}+\left(\frac{\dot{c}}{c}\right)^{\$}][\frac{1}{\varphi^{\$}}\frac{1}{z^{\$}}+\left(\frac{\dot{c}}{c}\right)^{\$}]+\frac{1}{\psi^{\$}}\frac{1}{\varphi^{\$}}\frac{1}{z^{\$}}\}^{-1}$.

The related eigenvector to λ_2, will be: $v_{12}=0$, v_{22}, $v_{32}=1$ and v_{42}, where v_{22} and v_{42} are given by equations: $(\dot{\psi}_{z})^{\$}v_{22}+(\dot{\psi}_{\psi})^{\$}-z^{\$}+\frac{\psi^{\$}}{(\varphi^{\$})^2}\frac{1}{z^{\$}}v_{42}=0$ and $(\dot{\varphi}_{z})^{\$}v_{22}-\frac{\varphi^{\$}}{(\psi^{\$})^2}+(\frac{1}{z^{\$}}\frac{1}{\varphi^{\$}}-z^{\$})v_{42}=0$. The related eigenvector to λ_3, will be: $v_{13}=v_{23}=0$, $v_{33}=1$ and $v_{43}=\frac{\varphi^{\$}}{(\psi^{\$})^2}(\frac{1}{\varphi^{\$}}\frac{1}{z^{\$}}-\lambda_3)^{-1}$. Finally, for λ_4 it will be: $v_{14}=v_{24}=0$, $v_{34}=1$ and $v_{44}=\frac{\varphi^{\$}}{(\psi^{\$})^2}(\frac{1}{\varphi^{\$}}\frac{1}{z^{\$}}-\lambda_4)^{-1}$.

Thus, it will be: $\begin{bmatrix}\tau\\ z\\ \psi\\ \varphi\end{bmatrix}=\begin{bmatrix}\tau^{\$}\\ z^{\$}\\ \psi^{\$}\\ \varphi^{\$}\end{bmatrix}+\begin{bmatrix}b_1e^{\lambda_1 t}\\ b_1v_{21}e^{\lambda_1 t}+b_2v_{22}e^{\lambda_2 t}\\ b_1v_{31}e^{\lambda_1 t}+b_2e^{\lambda_2 t}+b_3e^{\lambda_3 t}+b_4e^{\lambda_4 t}\\ b_1v_{41}e^{\lambda_1 t}+b_2v_{42}e^{\lambda_2 t}+b_3v_{43}e^{\lambda_3 t}+b_4v_{44}e^{\lambda_4 t}\end{bmatrix}$.

Transversality condition asks for: $\lim_{t\to\infty}\left(e^{-\rho t}\frac{\varphi}{\psi}\right)=0$. It is $\lambda_1-\rho<0$, $\lambda_2-\rho=z^{\$}-\rho>0$, $\lambda_3-\rho<0$ and one may prove that $\lambda_4-\rho=0.5[\sqrt{\Delta}-(z^{\$}-\rho)\frac{(1-\omega)+\alpha(2\omega-1)}{\alpha+2(1-\omega)(1-\alpha)}-\rho]>0$, i.e., for the transversality condition to be satisfied, it should be $b_2=b_4=0$. From initial conditions one gets: $b_1=\tau_0-\tau^{\$}$ & $b_3=[(\varphi_0-\varphi^{\$})-(\tau_0-\tau^{\$})v_{41}](v_{43})^{-1}$. Thus, it will finally be:

$$\tau=\tau^{\$}+(\tau_0-\tau^{\$})e^{-(\dot{c}/c)^{\$}t}$$

$$z=z^{\$}+(\tau_0-\tau^{\$})v_{21}e^{-(\dot{c}/c)^{\$}t}$$

$$\psi=\psi^{\$}+(\tau_0-\tau^{\$})v_{31}e^{-(\dot{c}/c)^{\$}t}+\frac{(\psi^{\$})^2}{\varphi^{\$}}(\frac{1}{\varphi^{\$}}\frac{1}{z^{\$}}-\lambda 3)[(\varphi_0-\varphi^{\$})-(\tau_0-\tau^{\$})v_{41}]e^{\lambda_3 t}$$

$$\varphi=\varphi^{\$}+(\tau_0-\tau^{\$})v_{41}e^{-(\dot{c}/c)^{\$}t}+[(\varphi_0-\varphi^{\$})-(\tau_0-\tau^{\$})v41]e^{\lambda_3 t}$$

F.

First Order Conditions for the given current value Hamiltonian will be:

$$\frac{\partial J}{\partial \tau}=0 \Leftrightarrow v_j[\frac{\partial w_j}{\partial \tau}+b_j\frac{\partial r_j}{\partial \tau}+\frac{r_f-r_j}{\theta}(\frac{\partial r_f}{\partial \tau}-\frac{\partial r_j}{\partial \tau})]+\mu_j c_j\frac{\partial r_j}{\partial \tau}+v_f[\frac{\partial w_f}{\partial \tau}+b_f\frac{\partial r_f}{\partial \tau}+\frac{r_j-r_f}{\theta}$$

$$(\frac{\partial r_j}{\partial \tau}-\frac{\partial r_f}{\partial \tau})]+\mu_f c_f\frac{\partial r_f}{\partial \tau}=0 \quad \textbf{(a)}$$

$$\frac{\partial J}{\partial b_j}=\rho v_j-\dot{v}_j \Leftrightarrow v_j[r_j+b_j\frac{\partial r_j}{\partial b_j}+\frac{\partial w_j}{\partial b_j}+\frac{r_f-r_j}{\theta}(\frac{\partial r_f}{\partial b_j}-\frac{\partial r_j}{\partial b_j})]+\mu_j c_j\frac{\partial r_j}{\partial b_j}+v_f[b_f\frac{\partial r_f}{\partial b_j}+\frac{\partial w_f}{\partial b_j}$$

$$+\frac{r_j-r_f}{\theta}(\frac{\partial r_j}{\partial b_j}-\frac{\partial r_f}{\partial b_j})]+\mu_f c_f\frac{\partial r_f}{\partial b_j}=\rho v_j-\dot{v}_j \quad \textbf{(b)}$$

$$\frac{\partial J}{\partial b_f}=\rho v_f-\dot{v}_f \Leftrightarrow v_j[b_j\frac{\partial r_j}{\partial b_f}+\frac{\partial w_j}{\partial b_f}+\frac{r_f-r_j}{\theta}(\frac{\partial r_f}{\partial b_{fj}}-\frac{\partial r_j}{\partial b_f})]+\mu_j c_j\frac{\partial r_j}{\partial b_f}+v_f[r_f+b_f\frac{\partial r_f}{\partial b_f}+\frac{\partial w_f}{\partial b_f}$$

$$+\frac{r_j-r_f}{\theta}(\frac{\partial r_j}{\partial b_f}-\frac{\partial r_f}{\partial b_f})]+\mu_f c_f\frac{\partial r_f}{\partial b_f}=\rho v_j-\dot{v}_j \quad \textbf{(c)}$$

$$\frac{\partial J}{\partial c_j}=\rho\mu_j-\dot{\mu}_j \Leftrightarrow \frac{1}{c_j}-v_j+\mu_j(r_j-\rho)=\rho\mu_j-\dot{\mu}_j \quad \textbf{(d)}$$

$$=\rho\mu_f-\dot{\mu}_f \Leftrightarrow \frac{1}{c_f}-v_f+\mu_f(r_f-\rho)=\rho\mu_f-\dot{\mu}_f \quad \textbf{(e)}$$

Remember (Technical Appendix A) that: $\frac{\partial k_j}{\partial \tau}=\frac{2}{\theta}\frac{\partial(r_j-r_f)}{\partial \tau}$ and $\frac{\partial k_f}{\partial \tau}=\frac{\partial k_j}{\partial \tau}$- and $\frac{G_j}{k_j}$ $=(A_j\tau)^{1/\alpha}[\frac{A_f}{A_j}\left(\frac{k_f}{k_j}\right)^{\alpha}]^{1/\alpha[1+(1-\alpha)(1-\omega_j-\omega_f)]}$. After some Algebra one ends up with: $\frac{\partial k_j}{\partial \tau}=\frac{2}{\theta}($ $\frac{y_j}{k_j}-\frac{y_f}{k_f})\frac{1-\alpha-\tau}{\tau}\{1+\frac{2}{\theta}\alpha(1-\alpha)(1-\tau)(\frac{y_j}{k_j}+\frac{y_f}{k_f})[1+(1-\alpha)(1-\omega_j-\omega_f)]^{-1}(\frac{1}{k_f}+\frac{1}{k_j})]\}^{-1}$

which in symmetry[1] becomes: $\frac{\partial k_j}{\partial \tau}=0$. This is a rather intuitive result: Changing the tax rate in both economies will not alter capital allocation.

In symmetry it will also be: $\frac{1}{G_j}\frac{\partial G_j}{\partial \tau}=\frac{1}{G_f}\frac{\partial G_{fj}}{\partial \tau}=\frac{1}{\alpha}\frac{1}{\tau}$ and $\frac{\partial r_j}{\partial \tau}=\frac{\partial r_f}{\partial \tau}=\frac{y}{k}\frac{1-\alpha-\tau}{\tau}$.

Finally, $\frac{\partial w_j}{\partial \tau} = \frac{\partial w_f}{\partial \tau} = y\frac{1-\alpha}{\alpha}\frac{1-\alpha-\tau}{\tau}$, hence: $\frac{\partial w_j}{\partial \tau} + k_j\frac{\partial r_j}{\partial \tau} = \frac{\partial w_f}{\partial \tau} + k_f\frac{\partial r_f}{\partial \tau} = y\frac{1-\tau-\alpha}{\alpha\tau}$.

Also in symmetry $b_j=k_j$ and $b_f=k_f$. Equation (a) now becomes: $vy\frac{1-\tau-\alpha}{\alpha\tau}+\mu c\frac{y}{k}\frac{1-\alpha-\tau}{\tau}+vy\frac{1-\tau-\alpha}{\alpha\tau}+\mu c\frac{y}{k}\frac{1-\alpha-\tau}{\tau}=0 \Leftrightarrow \frac{y}{k}\frac{1-\tau-\alpha}{\alpha\tau}(vk+\alpha\mu c)=0 \Leftrightarrow \tau=1-\alpha$.

In symmetry it will also be: $\frac{\partial r_j}{\partial b_j}=-\alpha(1-\tau)\frac{y}{k}(1-\alpha)(1-\omega)\frac{\theta}{2}\frac{1}{k}\{\frac{\theta}{2}[1-(1-\alpha)(2\omega-1)]+4\alpha(1-\alpha)(1-\tau)(1-\omega)\frac{y}{k}\frac{1}{k}\}^{-1}$, $\frac{\partial w_j}{\partial b_j}+b\frac{\partial r_j}{\partial b_j}=(1-\alpha)(1-\tau)\frac{y}{k}\{\frac{\theta}{2}[\alpha\omega+(1-\alpha)(1-\omega)]+2\alpha(1-\alpha)(1-\tau)(1-\omega)\frac{y}{k}\frac{1}{k}\}\{\frac{\theta}{2}[1-(1-\alpha)(2\omega-1)]+4\alpha(1-\alpha)(1-\tau)(1-\omega)\frac{y}{k}\frac{1}{k}\}^{-1}$ and $k\frac{\partial r_f}{\partial b_j}+\frac{\partial w_f}{\partial b_j}=(1-\alpha)(1-\tau)(1-\omega)\frac{y}{k}[\frac{\theta}{2}+2\alpha(1-\alpha)(1-\tau)\frac{y}{k}\frac{1}{k}]\{\frac{\theta}{2}[1+(1-\alpha)(1-2\omega)]+4\alpha(1-\alpha)(1-\tau)(1-\omega)\frac{y}{k}\frac{1}{k}\}^{-1}$.

Equation (b) now becomes: $\frac{\dot{v}}{v}=\rho-r-(1-\alpha)(1-\tau)\frac{y}{k}\{\frac{\theta}{2}[\alpha\omega+(1-\alpha)(1-\omega)]+2\alpha(1-\alpha)(1-\tau)(1-\omega)\frac{y}{k}\frac{1}{k}-(1-\omega)[\frac{\theta}{2}+2\alpha(1-\alpha)(1-\tau)\frac{y}{k}\frac{1}{k}]\}\{\frac{\theta}{2}[1-(1-\alpha)(2\omega-1)]+4\alpha(1-\alpha)(1-\tau)(1-\omega)\frac{y}{k}\frac{1}{k}\}^{-1} \Leftrightarrow \frac{\dot{v}}{v}=\rho+\delta-(1-\tau)\frac{y}{k}$. Finally, equation (c) yields to: $\frac{\dot{\mu}}{\mu}=\frac{v}{\mu}+\rho-\frac{1}{c}\frac{1}{\mu}-(r-\rho)$.

1. We assume that $\omega_{jj}=\omega_{ff}=\omega$, where $0<1-\omega\leq 0.5$