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GROWTH, WEALTH ACCUMULATION AND ENVIRONMENTAL CHANGE IN PORTFOLIO CHOICE AND TRADE

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Abstract
The main purpose of this paper is to study global economic growth against national capital accumulation and environmental change in conditions of free international trade. The paper addresses issues related to changes in inequalities of income, wealth and economic structures between countries. Another unique contribution is to include portfolio equilibrium along with physical wealth and other assets (such as gold and diamond) in the multi-country growth model. We develop a multi-country model of wealth and environment as endogenous variables, along with government intervention in environmental protection. Governments collect taxes from different sources of producers' outputs and consumers' income. We apply Zhang's alternative concept of disposable income and utility function to model behaviour of households. We build the dynamics of J-country world economy, the behaviour of which is described by differential equations. We simulate the movement of the 3-country global economy and carry out comparative dynamic analysis with regard to certain parameters.

JEL Classification: O41
Keywords: Global Economic Growth, Trade, Transboundary Pollution, Gold, Portfolio Equilibrium, Inequalities Between Countries

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1. Introduction

The world is well-connected, both economically and environmentally. The current pandemic shows how one occurrence in some unknown part of the world has caused global social and economic changes. Changes in savings and improved productivity in one region, such as East Asia, have great impact on the global economic geography. As economy has been becoming increasingly more complex, households deal with portfolio choices. It is obvious that pollution is no more a local or national concern. Economies with large populations, such as China and India, have serious implications on the global environment if their economies are further expanded. Low costs of transportation, fast flow of information, globalised education, and widely spread, culture-independent, rational, and scientific knowledge have changed how basic economic laws function globally. Price equalisation theories have become more valid, after one excludes frictions, such as trade barriers, various taxes, and tariffs. The world economy is becoming a village due to these changes. As the world becomes more closely interdependent, it is important to develop a global growth model that takes account of interdependence between these variables across countries and over time.

The main purpose of this paper is to study global economic growth against national capital accumulation and environmental change in conditions of free international trade. The paper addresses issues related to changes in inequalities of income, and wealth, and economic structures between countries. Another unique contribution is to include portfolio equilibrium along with physical wealth and other assets (such as gold and diamond) in the multi-country growth model. We develop a multi-country model of wealth and environment as endogenous variables along with government intervention in environmental protection. Governments collect taxes from different sources of producers' outputs and consumers' income.

A main character of many contemporary economies is that households hold many assets, such as housing, land, stocks, precious metals, gold, cash in different currencies, in their portfolios. The complexity of a portfolio is increased in many parts of the world (e.g., Uhler and Gragg, 1971; Agell and Edin, 1990; Cobb-Clark and Hilderbrand, 2009; Gaudecker, 2015). What was described by Guiso *et al.* (2002) is applicable to the current state of relevant literature on analysis of portfolio equilibrium: "Until recently, researchers in economics and finance paid relatively little attention to household portfolios. Reasons included the tendency of most households to hold simple portfolios, the inability of the dominant asset pricing models to account for household portfolio incompleteness, and the lack of detailed databases on household portfolios in many countries until the late 1980s or 1990s. Now, however, the analysis of household portfolios is emerging as a field of vigorous study." Although household portfolios are frequently modelled on microeconomic grounds, there are only a few theoretical growth models that explicitly take account of multiple assets and portfolio equilibrium. An original contribution of this paper is to introduce portfolio choice equilibrium into the neoclassical growth theory into the endogenous environment. For the purpose of modelling simplicity, portfolio choice refers to the decision on holding physical wealth and gold. It can be shown that we can easily generalise gold to include other assets, such as land, stocks, diamond, and the like (e.g., Zhang, 2020).

Economic growth and environmental changes have increasingly drawn attention in public debates and academic literature papers. Global warming is a main concern of many governments. It is easy, nowadays, to see that economic and environmental systems have a close interactional relationship. Rigorous economic theory still lacks a deep analysis of the significance of the environment in economic growth. It is crucial to study the economic system as an integrated whole rather than an unconnected subsystem. This study introduces endogenous environment into a multi-country growth model along with endogenous wealth and portfolio equilibrium. The environment is affected by production and consumption. Many studies have been conducted to examine the interdependence among economic growth, consumption, and the environment (for instance, Ploude, 1972; Forster, 1973; John and Pecchenino, 1994; Lamla, 2009; Prieur, 2009; Tsurumi and Managi, 2010; Gassebner *et al.* 2011; and Lin and Liscow, 2012). As pointed out by Fullerton and Kim (2008), existing research has proposed different models for analysing different questions in a non-integrated way. We make a contribution to the literature of interdependence between growth and the environment by examining growth, trade, and environmental change within an integrated framework.

We model capital mobility and trade on the basis of neoclassical growth theory. National economies are composed of Uzawa's two-sectors plus one environmental sector. We model the trade pattern on the Oniki-Uzawa trade model. We synthesise these approaches within an integrated framework by applying an alternative approach to consumer behaviour according to Zhang (1993, 2005). The model is a synthesis of two models recently proposed by Zhang. Zhang (2014) developed a global growth model on the basis of neoclassical growth theory. Zhang (2016) examined the value of gold and portfolio equilibrium conditions in the global market within a global growth equilibrium framework. This paper makes a synthesis of the main ideas in the two models. This paper is organised as follows. Section 2 defines the global growth model of portfolio equilibrium with endogenous changes in capital and the environment. Section 3 shows that the J-country global economy is described by 2J differential equations. Section 3 also simulates the model and depicts the movement of the global economy. Section 4 conducts a comparative dynamics analysis with regard to certain parameters. Section 5 concludes the study.

2. The global growth model based on wealth, the environment, and gold

This study is concerned with a global economic system composed of J national economies, indexed by $j = 1, ..., J$. Each national economy consists of three sectors: the capital goods sector, the consumer goods sector, and the environmental sector. National economies produce globally homogenous capital goods. We model the environment sector according to Zhang (2014). Each country's environment sector is financially supported by the national government. The national government collects taxes from producers and consumers. We model capital goods and consumer goods sectors on the Uzawa two-sector model (Uzawa, 1961; Burmeister and Dobell, 1970; Azariadis, 1993; Barro and Sala-i-Martin, 1995). Trade is described according to the Oniki-Uzawa model (Oniki and Uzawa, 1965). We add the environment sector to the Oniki-Uzawa trade model. The consumer goods sector is not globally tradable. Assets are owned by households. Saving is undertaken only by households. Households distribute disposable income proposed by Zhang (2005) to consume and save. All input factors are fully employed. It is assumed that there is a fixed amount of gold in the world. Gold is owned and used by households. Gold can be sold in free markets without any friction and transaction costs (Barro, 1979; Barsky and Summers, 1988; and Chappell and Dowd, 1997). The price of capital goods is a unit. We measure prices in terms of capital goods. The wage and interest rates are determined by their marginal values in perfectly competitive markets. We assume that trade is free. We omit possible transaction costs. This implies the interest rate is identical throughout the world economy. Capital goods and labour are used as input factors in the three sectors. We introduce:

N_j — country j's fixed population;
$K(t)$ — capital stocks of the world economy;
$K_j(t)$ and $\bar{K}_j(t)$ — total capital stock employed by country j and value of physical wealth owned by country j;
$E_j(t)$ — level of pollution in country;
i, s and e — subscript index standing for capital goods sector, consumer goods sector, and environment sector;
$N_{jm}(t)$ and $K_{jm}(t)$ — labour force and capital stocks employed by sector m in country j; $m = i, s, e$;
$F_{jm}(t)$ — output level of sector m in country j;
$w_j(t)$ and $p_j(t)$ — wage rate and price of consumer goods in country j;
$r(t)$ and $p_g(t)$ — (internationally equal) rate of interest and price of gold;
$\bar{k}_j(t)$ and $\bar{g}_j(t)$ — value of physical wealth and amount of gold owned by a-typical household in country j;
$c_j(t)$ — consumption level of consumer goods by the typical household in country j;
τ_{jm} and $\bar{\tau}_{jm}$ — fixed tax rate in sector m and $\bar{\tau}_{jm} \equiv 1 - \tau_{jm}, m = i, s$;

τ_{jw} and $\bar{\tau}_{jw}$ — tax rate on wage income in country j and $\bar{\tau}_{jw} \equiv 1 - \tau_{jw}$;
τ_{jc} — tax rate on consumption; and
δ_{kj} — fixed depreciation rate of capital in country j;

Capital goods sectors

Production of capital goods sector in country j is specified as follows:

$$F_{ji}(t) = A_{ji}\, \Gamma_{ji}(E_j)\, K_{ji}^{\alpha_{ji}}(t)\, N_{ji}^{\beta_{ji}}(t), A_{ji}, \alpha_{ji}, \beta_{ji} > 0, \alpha_{ji} + \beta_{ji} = 1, \quad (1)$$

where A_{ji}, α_{ji}, and β_{ji} are positive parameters. We use $\Gamma_{ji}(E_j)$ to describe how the environment affects the productivity of the capital goods sector. We will specify this function when simulating the model. The marginal conditions imply that

$$r(t) + \delta_{kj} = \frac{\alpha_{ji}\, \bar{\tau}_{ji}\, F_{ji}(t)}{K_{ji}(t)}, w_j(t) = \frac{\beta_{ji}\, \bar{\tau}_{ji}\, F_{ji}(t)}{N_{ji}(t)} \quad (2)$$

Consumer goods sectors

Production in the consumer goods sector is modelled as:

$$F_{js}(t) = A_{js}\, \Gamma_{js}\left(E_j(t)\right)\, K_{js}^{\alpha_{js}}(t)\, N_{js}^{\beta_{js}}(t), \alpha_{js} + \beta_{js} = 1, \alpha_{js}, \beta_{js} > 0, \quad (3)$$

where A_{js}, α_{js}, and β_{js} are parameters and $\Gamma_{js}(E_j(t))$ is a function of the environment. The marginal conditions are given as:

$$r(t) + \delta_k = \frac{\alpha_{js}\, \bar{\tau}_{js}\, p_j(t)\, F_{js}(t)}{K_{js}(t)}, w_j(t) = \frac{\beta_{js}\, \bar{\tau}_{js}\, p_j(t)\, F_{js}(t)}{N_{js}(t)}. \quad (4)$$

Modelling change in the environment

Following Zhang (2014), we specify changes in the environment as follows:

$$\dot{E}_j(t) = \theta_{ji}\, F_{ji}(t) + \theta_{js}\, F_{js}(t) + \theta_j\, C_j(t) - F_{je}(t) - \bar{\theta}_j\, E_j(t) + \Omega_j\left(\left(E_q(t)\right)\right), \quad (5)$$

in which θji, θjx, θj, and $\bar{\theta}_i$ are positive parameters and

$$F_{je}(t) = A_{je}\, \Gamma_{je}\left(E_j(t)\right)\, K_{je}^{\tilde{\alpha}_{je}}(t)\, N_{e}^{\tilde{\beta}_{je}}(t), A_{je}, \tilde{\alpha}_{je}, \tilde{\beta}_{je} > 0, \quad (6)$$

where Aje αje, and βje are positive parameters, and $\Gamma_{je}\left(E_j(t)\right) (\geq 0)$ is a function of $E_j(t)$. We use $\theta_{ji}F_{ji}$ to model pollutant emission during production processes. It is assumed that emission is linearly positively proportional to the output level (Gutiérrez, 2008). Parameter θ_{ji} implies that in consuming one unit of the goods quantity θ_{ji} is left as waste. We use $\theta_{jm}F_{jm}$ to measure pollutants emitted by sector θ_{jm}. Emission of pollutants by consumers is $\theta_j C_j$. We use parameter $\bar{\theta}_j$ to measure rate of natural purification. Term $\bar{\theta}_j E_j$ measures the rate at which nature purifies the environment. We use $K_{je}^{\widetilde{\alpha}_e} N_{je}^{\widetilde{\beta}_e}$ to reflect that the purification rate of the environment sector is positively related to capital and labour inputs. Term Γ_{je} means that purification efficiency is related to stock pollutants. We take account of possible pollution of country j for all countries with $\Omega_j((E_q))$. Transboundary pollution has been well-studied in literature on international pollution (e.g., Copeland and Taylor, 1994; Ono, 1998; Schweirger and Woodland, 2008; and Suhardiman and Giordano, 2012).

Portfolio choice equilibrium with physical wealth and gold

We model portfolio choice equilibrium against physical wealth and gold according to Zhang (2016). Gold is owned only by households. Gold is sold and bought in free markets. There is neither friction nor transaction cost in markets for gold. There is neither loss nor depreciation of gold. Households choose physical wealth and gold. To model equilibrium conditions for gold, we assume that households can 'rent' it in free markets for 'decoration' use. The representative household gold is used either for decoration or rented to other households. We represent rent of gold by $R_g(t)$ in global markets. Consider now an investor with one unit of money. S/he can either invest in capital goods, thereby, earning profit equal to the net own-rate of return or invest in gold, thereby, earning profit equal to the net own-rate of return $R_g(t)/p_g(t)$. As capital and gold markets are at portfolio equilibrium in free markets, two options must yield equal returns, i.e.

$$\frac{R_g(t)}{p_g(t)} = r(t). \tag{7}$$

Consumer behaviours

This study applies Zhang's utility function and concept of disposable income to analyse behaviour of households (Zhang, 1993; 2005). There are three variables for households to decide on consumption levels of goods and gold, and on how much to save. The total value of wealth owned by a household $a_j(t)$is the sum of two assets' values:

$$a_j(t) = \bar{k}_j(t) + p_g(t)\, \bar{g}_j(t). \tag{8}$$

The household's current income from interest payments, wage payments, and gold interest income $R_g(t)\bar{g}_j(t)$ is:

$$y_j(t) = r(t)\, \bar{k}_j(t) + \bar{\tau}_{jw}\, w_j(t) + R_g(t)\, \bar{g}_j(t).$$

The per capita disposable income is the sum of the current disposable income and the value of wealth. We have:

$$\hat{y}_j(t) = y_j(t) + a_j(t). \tag{9}$$

The disposable income is distributed between saving and consumption. Saving $s_j(t)$ originates from disposable income. We have the budget constraint:

$$\left(1+\tau_{jc}\right) p_j(t)\, c_j(t) + R_g(t)\, \hat{g}_j(t) + s_j(t) = \hat{y}_j(t). \tag{10}$$

The household decides on $s_j(t)$, $c_j(t)$, and $\hat{g}_j(t)$. Zhang's utility function is as follows:

$$U_j(t) = \Gamma_j\left(E_j(t)\right)\, c_j^{\xi_{0j}}(t)\, \hat{g}_j^{\gamma_{0j}}(t)\, s_j^{\lambda_{0j}}(t), \xi_{0j}, \gamma_{0j}, \lambda_{0j} > 0, \tag{11}$$

where $\Gamma_j\left(E_j(t)\right)$ is a function related to the environment, ξ_{0j} is the propensity to consume goods, γ_{0j} is the propensity to use gold, and λ_{0j} the propensity to own wealth. It should be noted that some studies examine how the environment directly affects a household's decision (e.g., Selden and Song, 1995; Balcao, 2001; Nakada, 2004; and Munro, 2009).

Household maximises subject to budget constraint (10) yields

$$p_j(t)\, c_j(t) = \xi_j\, \hat{y}_j(t), R_g(t)\, \hat{g}_j(t) = \gamma_j\, \hat{y}_j(t), s_j(t) = \lambda_j\, \hat{y}_j(t), \tag{12}$$

where

$$\xi_j \equiv \frac{\rho_j\, \xi_{0j}}{1+\tau_{jc}}, \gamma_j \equiv \rho_j\, \gamma_{0j}, \lambda_j \equiv \rho_j\, \lambda_{0j}, \rho_j \equiv \frac{1}{\xi_{0j}+\gamma_{0j}+\lambda_{0j}}.$$

Change in wealth

The change in a household's wealth is equal to savings minus dissaving, that is

$$\dot{a}_j(t) = s_j(t) - a_j(t). \tag{13}$$

Household own all gold

Globally, households own all the gold:

$$\sum_{j=1}^{J} \bar{g}_j(t)\bar{N}_j = G. \tag{14}$$

Gold is fully utilised

The total amount of gold used for 'decoration' equals the total gold in markets

$$\sum_{j=1}^{J} \hat{g}_j(t)\bar{N}_j = G. \tag{15}$$

The environment sector's input factors

The government uses up tax income on employing labour force and capital stock. The government combines taxes from producers and households. Government tax income is:

$$Y_{je}(t) = \tau_{ji} F_{ji}(t) + \tau_{js} F_{js}(t) + \tau_{jc} c_j(t) N_j + \tau_{jw} w_j(t) N_j. \tag{16}$$

The government budget means:

$$(r(t) + \delta_k)K_{je}(t) + w_j(t) N_{je}(t) = Y_{je}(t). \tag{17}$$

The government employs labour force and capital stocks to ensure that the purification rate is maximised under budget constraints. The government's problem is formulated as

$$\underset{\{K_{je}(t),N_{je}(t)\}}{\text{Max}} F_e(t)$$
$$\text{s.t.: (17).}$$

Marginal conditions imply:

$$(r(t) + \delta_k)K_{je}(t) = \alpha_{je} Y_{je}(t), w_j(t) N_{je}(t) = \beta_{je} Y_{je}(t), \tag{18}$$

in which

$$\alpha_{je} \equiv \frac{\tilde{\alpha}_{je}}{\tilde{\alpha}_{je} + \tilde{\beta}_{je}}, \beta_{je} \equiv \frac{\tilde{\beta}_{je}}{\tilde{\alpha}_{je} + \tilde{\beta}_{je}}.$$

Demand and supply

The equilibrium condition for consumer goods implies:

$$c_j(t) N_j = F_{js}(t), j = 1, \dots, J. \tag{19}$$

National labour and capital are fully employed:

$$K_{ji}(t) + K_{js}(t) + K_{je}(t) = K_j(t), \; N_{ji}(t) + N_{js}(t) + N_{je}(t) = N_j. \tag{20}$$

The value of global capital stock equals the value of global physical wealth

$$K(t) = \sum_{j=1}^{J} K_j(t) = \sum_{j=1}^{J} \bar{k}_j(t) N_j . \tag{21}$$

The change in global capital equals total capital output minus global capital depreciation

$$\dot{K}(t) = F(t) - \sum_{j=1}^{J} \delta_{kj} K_j(t) . \tag{22}$$

where

$$F(t) = \sum_{j=1}^{J} F_j(t).$$

The model is completed. From a structural point of view, the model is general in the sense that some well-known models in economic theory, such as the Solow growth model (Solow, 1956), Uzawa's two sector model (Uzawa, 1961), and Oniki-Uzawa trade model (Oniki and Uzawa, 1965), can be considered as its special cases.

3. Global dynamics

As the global economy is composed of any (finite) number of national economies and there is free trade between countries, the dynamic system is highly dimensional. We show properties of the system by simulation. The following lemma shows that the dimension of the dynamic system is twice the number of countries. We provide a computational procedure for calculating all variables at any point in time. We define a new variable $z_1(t)$:

$$z_1(t) \equiv \frac{r(t) + \delta_{1k}}{w_1(t)}.$$

Lemma

The dynamics of the world economy is determined by $2J$ differential equations with $z_1(t)$, $\{\alpha_j(t)\}$ and $(E_j(t))$ where $\{\alpha_j(t)\} = (\alpha_2(t), ..., \alpha_J(t))$ and $(E_j(t)) = (E_1(t), ... , E_J(t)$ as variables:

$$\dot{z}_1(t) = \Phi_1\left(z_1(t), \left(E_j(t)\right), \{a_j(t)\}\right),$$
$$\dot{a}_j(t) = \Phi_j\left(z_1(t), \left(E_j(t)\right), \{a_j(t)\}\right), j = 2, \ldots, J,$$
$$\dot{E}_j(t) = \Omega_j\left(z_1(t), \left(E_j(t)\right), \{a_j(t)\}\right), j = 1, \ldots, J, \tag{23}$$

in which Φ_j and Ω_j are unique functions of the $2J$ variables. The functions are provided in the Appendix. Moreover, the other variables are uniquely determined as functions of the $2J$ variables through the following procedure: $r(t)$ and $w_j(t)$ with (A2) → $p_j(t)$ from (A4) → $a_1(t)$ by (A20) → $K_j(t)$ by (A17) → $N_{ji}(t)$ and $N_{je}(t)$ from (A11) → $N_{js}(t)$ in (A7) → $K_{je}(t)$, $K_{js}(t)$, and $K_{ji}(t)$ in (A1) → $\hat{y}_j(t)$ from (A5) → $F_{ji}(t)$, $F_{js}(t)$ and $F_{je}(t)$ by the definitions → $c_j(t)$ and $s_j(t)$ from (12) → $Y_{je}(t) = w_j(t)\, N_{je}(t) / \beta_{je}$ → $K(t) = \Sigma_j K_j(t)$ → $R_g(t)$ by (A21) → $p_g(t)$ by (7) → $\bar{g}_j(t)$ by (12).

We can follow the computational procedure to show the movement of the global economy with any number of countries. We now examine a -country global economy by simulation. As in Zhang (2014), the functions related to environmental quality are specified

$\Gamma_{jm}\left(E_j(t)\right) = E_j^{-b_{jm}}(t), \Gamma_j\left(E_j(t)\right) = E_j^{-b_j}(t), j = 1,2,3, m = i, s, e.$ undary pollution functions are specified as:

$$\Omega_q\left(\left(E_q(t)\right)\right) = \sum_{j,j\neq q}^{J} \theta_{jq}\, E_j(t).$$

We require $\theta_{jq} \geq 0.$ The transboundary pollution functions mean that any country's environment may be affected by all other countries. We consider the transboundary pollution rate is linearly related to pollutant source countries. We specify the parameters as follows:

$$\begin{pmatrix} N_1 \\ N_2 \\ N_3 \end{pmatrix} = \begin{pmatrix} 3 \\ 10 \\ 30 \end{pmatrix}, \begin{pmatrix} A_{1i} \\ A_{2i} \\ A_{3i} \end{pmatrix} = \begin{pmatrix} 1.7 \\ 1 \\ 0.8 \end{pmatrix}, \begin{pmatrix} A_{1s} \\ A_{2s} \\ A_{3s} \end{pmatrix} = \begin{pmatrix} 1.5 \\ 0.9 \\ 0.7 \end{pmatrix}, \begin{pmatrix} A_{1e} \\ A_{2e} \\ A_{3e} \end{pmatrix} = \begin{pmatrix} 1.2 \\ 1 \\ 0.9 \end{pmatrix}, \begin{pmatrix} \lambda_{10} \\ \lambda_{20} \\ \lambda_{30} \end{pmatrix} = \begin{pmatrix} 0.6 \\ 0.55 \\ 0.5 \end{pmatrix},$$

$$\begin{pmatrix} b_{1i} \\ b_{2i} \\ b_{3i} \end{pmatrix} = \begin{pmatrix} b_{1s} \\ b_{2s} \\ b_{3s} \end{pmatrix} = \begin{pmatrix} b_{1e} \\ b_{2e} \\ b_{3e} \end{pmatrix} = \begin{pmatrix} 0.1 \\ 0.02 \\ 0.01 \end{pmatrix}, \begin{pmatrix} \bar{\theta}_1 \\ \bar{\theta}_2 \\ \bar{\theta}_3 \end{pmatrix} = \begin{pmatrix} 0.08 \\ 0.12 \\ 0.11 \end{pmatrix}, \begin{pmatrix} \tau_{1w} \\ \tau_{2w} \\ \tau_{3w} \end{pmatrix} = \begin{pmatrix} 0.01 \\ 0.02 \\ 0.02 \end{pmatrix},$$

$$\begin{pmatrix} \xi_{0j} \\ \tau_{jc} \\ \tau_{ji} \end{pmatrix} = \begin{pmatrix} 0.2 \\ 0.01 \\ 0.01 \end{pmatrix}, \begin{pmatrix} \tau_{js} \\ \tilde{\alpha}_{je} \\ \tilde{\beta}_{je} \end{pmatrix} = \begin{pmatrix} 0.01 \\ 0.4 \\ 0.2 \end{pmatrix}, \begin{pmatrix} b_j \\ \theta_{ji} \\ \theta_j \end{pmatrix} = \begin{pmatrix} 0.01 \\ 0.08 \\ 0.03 \end{pmatrix}, \begin{pmatrix} \delta_{1k} \\ \delta_{2k} \\ \delta_{3k} \end{pmatrix} = \begin{pmatrix} 0.05 \\ 0.04 \\ 0.04 \end{pmatrix},$$

$$\alpha_{ji} = 0.31,\ \alpha_{js} = 0.33,\ G = 1,\ \gamma_{0j} = 0.005,\ \theta_{js} = 0.1,\ \theta_{jq} = 0.01,\ j,\ q = 1,2,3. \tag{24}$$

We fix the total amount of gold in the world as one unit. The populations are respectively 3,3 and 30 in order. Country 3 has the largest population. The total productivity factor of the capital goods sectors is, respectively: 1.7, 1 and 0.8. The total productivity factor of the consumer goods sectors is, respectively 1.5, 0.9 and 0.7. The total productivity factor of the environment sectors is, respectively 1.2, 1 and 0.9. Country 1's propensity to save is the highest; country 2's comes next and country 3's is the lowest. Tax rates on consumption level are one percent. Tax rates on production sectors and interest income of wealth are one or two percent. We simulate the model with the following initial conditions:

$z_1(0) = 0.12,\ \alpha_2(0) = 3,\ \alpha_3(0) = 2.3,\ E_1(0) = 10.5,\ E_2(0) = 10,\ E_3(0) = 55.$

The motion of the variables is plotted in Figure 1. In Figure 1, the global output is:

$$Y(t) = \sum_j \left\{F_{ji}(t) + p_j(t)\, F_{js}(t)\right\}.$$

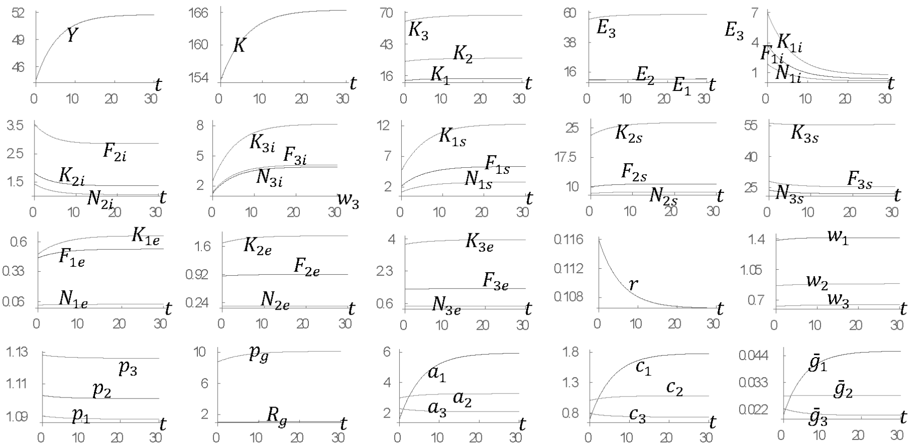

Figure 1. Motion of World Economy

The global output and capital stock are enhanced from the initial state. National economies employ more physical capital. Sectors in different countries experience different paths of economic development. The capital goods sectors of countries 1 and 2 produce less output and employ less the two input factors. The capital goods sector of country 3 produces more output and employs more the two input factors. The consumer goods sector of countries 1 and 2 produce more output and employ more the two input factors. The consumer goods sector of country 3 produces less output and employs less the two input factors. The three economies also experience different paths of environmental changes. Country 3's environment deteriorates. The other two countries' environmental conditions are slightly changed. The rate of interest falls. The price of gold rises and the rent of gold changes slightly. All the wage rates are increased. All service charges are lowered. Country 1's consumption levels of goods and gold and level of wealth are increased. Country 2's consumption levels of goods and gold and level of wealth per household change slightly. Country 3's consumption levels of goods and gold and level of wealth per household are reduced.

From Figure 1, we see that the system is approaching a stationary state over time. This implies the existence of an equilibrium point. We identify the values of an equilibrium point

$K = 166.5, Y = 51.8, r = 0.107, p_g = 10.08, R_g = 1.08, \bar{g}_1 = 0.046 ,, \bar{g}_2 = 0.028 ,$
$\bar{g}_3 = 0.02.$

$$\begin{pmatrix} E_1 \\ E_2 \\ E_3 \end{pmatrix} = \begin{pmatrix} 11.083 \\ 10.77 \\ 58.63 \end{pmatrix}, \begin{pmatrix} Y_{1e} \\ Y_{2e} \\ Y_{3e} \end{pmatrix} = \begin{pmatrix} 0.15 \\ 0.41 \\ 0.87 \end{pmatrix}, \begin{pmatrix} p_1 \\ p_2 \\ p_3 \end{pmatrix} = \begin{pmatrix} 1.1 \\ 1.09 \\ 1.13 \end{pmatrix}, \begin{pmatrix} w_1 \\ w_2 \\ w_3 \end{pmatrix} = \begin{pmatrix} 1.41 \\ 0.89 \\ 0.65 \end{pmatrix}, \begin{pmatrix} F_{1i} \\ F_{2i} \\ F_{3i} \end{pmatrix} = \begin{pmatrix} 0.41 \\ 1.37 \\ 3.9 \end{pmatrix},$$

$$\begin{pmatrix} F_{1s} \\ F_{2s} \\ F_{3s} \end{pmatrix} = \begin{pmatrix} 5.33 \\ 10.83 \\ 22.13 \end{pmatrix}, \begin{pmatrix} F_{1e} \\ F_{2e} \\ F_{3e} \end{pmatrix} = \begin{pmatrix} 0.53 \\ 0.92 \\ 1.39 \end{pmatrix}, \begin{pmatrix} N_{1i} \\ N_{2i} \\ N_{3i} \end{pmatrix} = \begin{pmatrix} 0.2 \\ 1.06 \\ 4.11 \end{pmatrix}, \begin{pmatrix} N_{1s} \\ N_{2s} \\ N_{3s} \end{pmatrix} = \begin{pmatrix} 2.77 \\ 8.79 \\ 25.5 \end{pmatrix},$$

$$\begin{pmatrix} N_{1e} \\ N_{2e} \\ N_{3e} \end{pmatrix} = \begin{pmatrix} 0.04 \\ 0.15 \\ 0.45 \end{pmatrix}, \begin{pmatrix} K_{1i} \\ K_{2i} \\ K_{3i} \end{pmatrix} = \begin{pmatrix} 0.8 \\ 2.88 \\ 8.18 \end{pmatrix}, \begin{pmatrix} K_{1s} \\ K_{2s} \\ K_{3s} \end{pmatrix} = \begin{pmatrix} 12.25 \\ 26.27 \\ 55.58 \end{pmatrix}, \begin{pmatrix} K_{1e} \\ K_{2e} \\ K_{3e} \end{pmatrix} = \begin{pmatrix} 0.65 \\ 1.86 \\ 3.97 \end{pmatrix}$$

$$\begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} = \begin{pmatrix} 5.93 \\ 3.27 \\ 2.1 \end{pmatrix}, \begin{pmatrix} c_1 \\ c_2 \\ c_3 \end{pmatrix} = \begin{pmatrix} 1.78 \\ 1.08 \\ 0.74 \end{pmatrix}.$$

We calculate the six eigenvalues at the equilibrium point:

$$-0{,}22, -0.2, -0{,}18, -0.13, -0.12, -0.08.$$

The equilibrium point is locally stable. This result is important as it confirms the validity of comparative dynamic analysis.

4. Comparative Dynamic Analysis

We followed the movement of the global economy. We now deal with how any exogenous changes affect global economy and national economies. We define a variable $\bar{\Delta} x(t)$, which represents the change rate of variable $x(t)$ in percentage due to changes in the specified parameter.

4.1. Country 1's propensity to use gold rises

We are first concerned with how the global economy is affected when country 1's propensity to use gold rises as follows: γ_{01}:0.005 ⇒ 0.007. The simulation result is plotted in Figure 2. As country 1 increases its preference for gold for decoration, the price and rent of gold are enhanced. Country 1 uses more gold, while the other two economies use less. Prices of consumer goods fall. Country 1 consumes less and has less wealth, while the other two economies' consumption levels and levels of wealth are affected slightly. The Global output and physical capital are reduced in association with falling wage rates and rising rates of interest. Environmental qualities are improved in the three economies. All economies employ less capital. Economic structural changes are plotted in Figure 2.

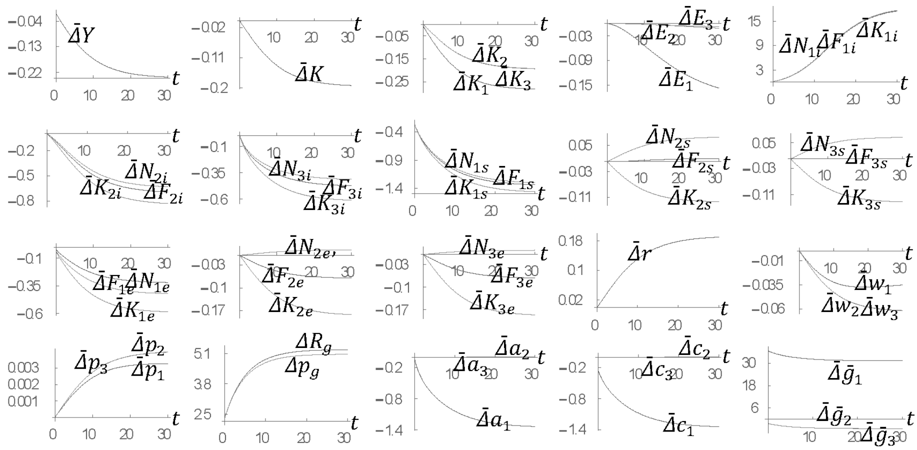

Figure 2. Country 1's Propensity to Use Gold Rises

4.3. Environmental tax rate in country 1's capital goods sector rises

We study impact of a rise in the environmental tax rate in country 1's capital goods sector as follows: τ1i:0.01 ⇒ 0.02. The simulation result is plotted in Figure 3. Country 1's environment sector expands. Environmental qualities are improved in all three economies. Country 1's capital goods sector shrinks. Global income and wealth rise initially and fall in the long term. Gold price and rent rise initially and fall in the long term. Country 1's household initially has more wealth, consumes more, and utilises more gold, but in the long term it has less wealth, consumes less, and utilises less gold. The behaviour of the other countries' households is slightly affected. The rate of interest falls initially and changes slightly in the long term. Country 1 has lower wage rates and lower prices of consumer goods.

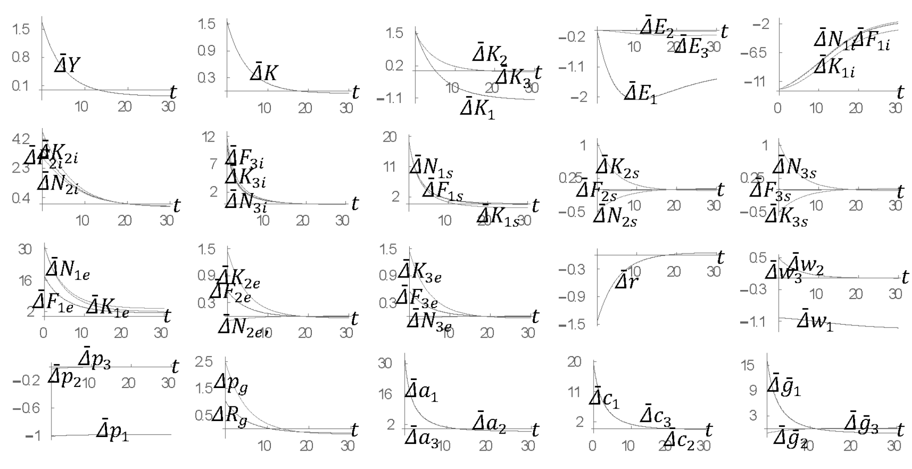

Figure 3. The Environmental Tax Rate in Country 1's Capital Goods Sector Rises

4.4. Country 3 increases consumption tax rate

We now study what happens in the global economy if the UE economy raises consumption tax rate as follows: τ3c:0.01 ⇒ 0.03. The simulation result is plotted in Figure 3. Country 3's consumption of consumer goods falls and country 3's consumer goods sector shrinks. All capital goods sectors expand. Global income rises, but global capital stock falls. All national economies have a better environment. Countries 1 and 2 employs more capital, while country 3 employs less capital. Gold price and gold rent initially fall, and prices of consumer goods fall in all economies. The rate of interest falls, and wage rates rise. In the long term environment sectors increase output levels.

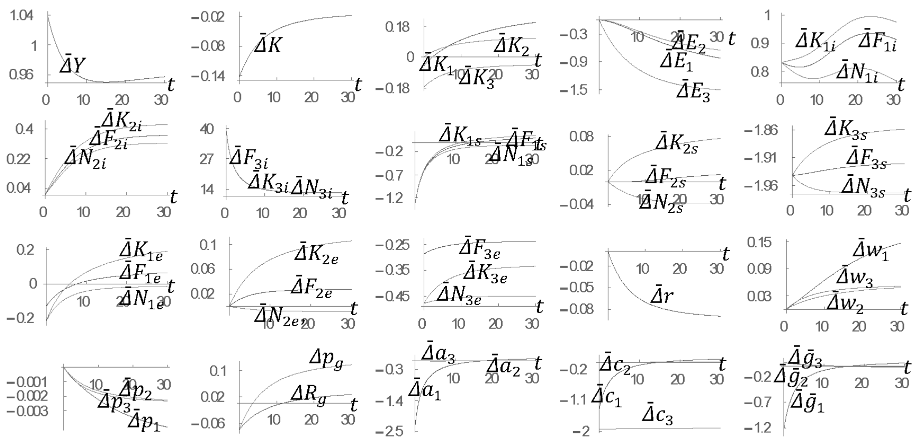

Figure 4. Country 3 increases Consumption Tax Rate

5. Concluding remarks

This paper studies global economic growth against national capital accumulation and environmental change in free international trade. It addresses issues related to changes in inequalities in income and wealth and economic structures between countries. Another unique contribution is made by introducing portfolio equilibrium between physical wealth and other assets (such as gold and diamond) in a multi-country growth model. The paper constructs a multi-country growth model with portfolio equilibrium between physical wealth and gold. The multi-country model treats wealth and the environment as endogenous variables along with government intervention in environmental protection. Governments collect taxes from different sources of producers' outputs and consumers' income. The study deals not only with issues related to changes in inequalities in income and wealth and economic structures between countries, but also differences in environmental changes between countries. We built the dynamics of a J-country world economy of which behaviour is described by $2J$ differential equations. We simulated the movement of the 3-country global economy and carried out comparative dynamic analysis with regards to certain parameters. As the model is composed of dynamic interdependence among many variables, we get relations among these variables which cannot be obtained from partial analyses. In the paper, for instance, we show that an improvement in one country's environmental policy will improve all countries' environment quality but reduce the country's economic activities. The country also consumes less and has lower capital stock and lower wage rates. This might explain why some countries are reluctant to make contributions to environmental improvement through enhancing tax rates on firms. We also examine effects of changes in countries' preferences and technologies. As the analytical framework is robust, it can provide different insights into the complexity of modern economic issues.

We can examine some important issues related to the interdependence between growth and environmental change in a comprehensive manner, not only because our theory is built on the economic mechanisms, but also because we gave the computational procedure for plotting dynamic paths of the nonlinear dynamic system. Although our modelling framework is truly comprehensive, it is a simplified analytical framework and can be extended and generalised in different directions. For instance, other forms of production or utility functions may be applied to the economic system. Transboundary pollution may occur in other ways. There are different ways to tax firms and households (e.g., Zhang, 2020).

Appendix: Proving the Lemma

Equations (2), (4) and (18) imply:

$$z_j \equiv \frac{r+\delta_k}{w_j} = \frac{N_{jm}}{\bar{\beta}_{jm} K_{jm}}, j = 1, \ldots, J, m = i, s, e, \tag{A1}$$

in which $\bar{\beta}_{jm} \equiv \beta_{jm}/\alpha_{jm}$. Inserting (A1) in (2) yields:

$$r = \alpha_{jr}\, \Gamma_{ji}\, z_j^{\beta_{ji}} - \delta_k, w_j = \alpha_j\, \Gamma_{ji}\, z_j^{-\alpha_{ji}}, \tag{A2}$$

in which

$$\alpha_{jr} = \alpha_{ji}\, \bar{\tau}_{ji}\, \bar{\beta}_{ji}^{\beta_{ji}}\, A_{ji}, \alpha_j = \frac{\beta_{ji}\, \bar{\tau}_{ji}\, A_{ji}}{\bar{\beta}_{ji}^{\alpha_{ji}}}.$$

Equation (A2) means:

$$r = \alpha_{jr}\, \Gamma_{ji}\, z_j^{\beta_{ji}} - \delta_{jk} = \alpha_{1r}\, \Gamma_{1i}\, z_1^{\beta_{1i}} - \delta_{1k}, j = 1, \ldots., J.$$

We solve the above equations

$$z_j\left(z_1, \left(E_j\right)\right) = \left(\frac{\alpha_{1r}\, \Gamma_{1i}\, z_1^{\beta_{1i}} + \delta_{jk} - \delta_{1k}}{\alpha_{jr}\Gamma_{ji}}\right)^{1/\beta_{ji}}, j = 2, \ldots., J. \tag{A3}$$

We, thus, solved r, w_j, and z_j, as functions of z_1 and (E_j). Equations (3) and (4) mean:

$$p_j\left(z_1,(E_j)\right)=\frac{\bar{\beta}_{js}^{\alpha_{js}}\, z_j^{\alpha_{js}}\, w_j}{\beta_{js}\,\bar{\tau}_{js}\,A_{js}\,\Gamma_{js}}. \tag{A4}$$

By (7)-(9), we solve:

$$\hat{y}_j=(1+r)a_j+\bar{\tau}_{jw}\,w_j. \tag{A5}$$

Substitute $p_j\, c_j=\xi_j\,\hat{y}_j$ into (19):

$$\xi_j\,N_j\,\hat{y}_j=p_j\,F_{js}. \tag{A6}$$

Insert (A5) in (A6):

$$N_{js}=\theta_j\,\bar{k}_j+\bar{\theta}_j, \tag{A7}$$

in which we apply $w_j\,N_{js}=\beta_{js}\,\bar{\tau}_{js}\,p_j\,F_{js}$ and:

$$\theta_j\left(z,(E_j)\right)\equiv\left(\frac{1+r}{w_j}\right)\beta_{js}\bar{\tau}_{js}\,\xi_j\,N_j,\bar{\theta}_j\equiv\beta_{js}\,\bar{\tau}_{js}\,\bar{\tau}_{jw}\,\xi_j\,N_j.$$

By (A1) and (16) we solve:

$$\frac{N_{ji}}{\bar{\beta}_{ji}}+\frac{N_{js}}{\bar{\beta}_{js}}+\frac{N_{je}}{\bar{\beta}_{je}}=z_jK_j. \tag{A8}$$

Substitute (A7) into (A8)

$$\frac{N_{ji}}{\bar{\beta}_{ji}}+\frac{N_{je}}{\bar{\beta}_{je}}=z_j\,K_j-\frac{\theta_j\,a_j}{\bar{\beta}_{js}}-\frac{\bar{\theta}_j}{\bar{\beta}_{js}}. \tag{A9}$$

Substituting (A7) into $N_{ji}+N_{js}+N_{je}=N_j$ yields:

$$N_{ji}+N_{je}=N_j-\theta_j\,a_j-\bar{\theta}_j. \tag{A10}$$

We solve (A9) and (A10) with N_{js} and N_{ji} as variables:

$$N_{ji}=a_{ji}+\tilde{b}_{ji}a_j-\bar{\beta}_jz_jK_j,N_{je}=a_{je}+\tilde{b}_{je}a_j+\bar{\beta}_jz_jK_j, \tag{A11}$$

where

$$a_{ji}\equiv\left(\frac{N_j-\bar{\theta}_j}{\bar{\beta}_{je}}+\frac{\bar{\theta}_j}{\bar{\beta}_{js}}\right)\bar{\beta}_j,\tilde{b}_{ji}\left(z,(E_j)\right)\equiv\left(\frac{1}{\bar{\beta}_{js}}-\frac{1}{\bar{\beta}_{je}}\right)\bar{\beta}_j\,\theta_j,$$

$$a_{je}\equiv-\left(\frac{\bar{\theta}_j}{\bar{\beta}_{js}}+\frac{N_j-\bar{\theta}_j}{\bar{\beta}_{ji}}\right)\bar{\beta}_j,\tilde{b}_{je}\left(z,(E_j)\right)\equiv\left(\frac{1}{\bar{\beta}_{ji}}-\frac{1}{\bar{\beta}_{js}}\right)\bar{\beta}_j\,\theta_j,\bar{\beta}_j\equiv\left(\frac{1}{\bar{\beta}_{je}}-\frac{1}{\bar{\beta}_{ji}}\right)^{-1}.$$

Insert (A1) in (2) and (5)

$$F_{ji} = \frac{A_{ji}\,\Gamma_{ji}\,N_{ji}}{\bar{\beta}_{ji}^{\alpha_{ji}}\,z_j^{\alpha_{ji}}},\; F_{js} = \frac{A_{js}\,\Gamma_{js}\,N_{js}}{\bar{\beta}_s^{\alpha_s}\,z_j^{\alpha_{js}}}. \tag{A12}$$

From (A12) and (12) we get:

$$Y_{je} = \Lambda_{ji}\,N_{ji} + \Lambda_{js}\,N_{js} + \tau_{jc}\,c_j\,N_j + \tau_{jw}\,w_j\,N_j, \tag{A13}$$

where

$$\Lambda_{ji}\left(z_1,(E_j)\right) \equiv \frac{\tau_{ji}\,A_{ji}\,\Gamma_{ji}}{\bar{\beta}_{ji}^{\alpha_{ji}}\,z_j^{\alpha_{ji}}},\; \Lambda_{js}\left(z_1,(E_j)\right) \equiv \frac{\tau_{js}\,A_{js}\,\Gamma_{js}}{\bar{\beta}_{js}^{\alpha_{js}}\,z_j^{\alpha_{js}}}.$$

By $p_j\,c_j = \xi_j\,\hat{y}_j$ and (A5), we solve:

$$c_j = \left(\frac{1+r}{p_j}\right)\xi_j\,a_j + \frac{\bar{\tau}_{jw}\,\xi_j\,w_j}{p_j}. \tag{A14}$$

From (A14) and (A13) we get:

$$Y_{je} = \bar{\Lambda}_j + \Lambda_{ji}\,N_{ji} + \Lambda_{js}\,N_{js} + \Lambda_j\,a_j, \tag{A15}$$

where

$$\Lambda_j\left(z_1,(E_j)\right) \equiv \left(\frac{1+r}{p_j}\right)\xi_j\,\tau_{jc}\,N_j,\; \bar{\Lambda}_j\left(z_1,(E_j)\right) \equiv \left(\frac{\bar{\tau}_{jw}\,\xi_j\,\tau_{jc}}{p_j} + \tau_{jw}\right)w_j\,N_j.$$

Substitute (A15) into $w_j\,N_{je} = \beta_{je}\,Y_{je}$

$$\frac{w_j\,N_{je}}{\beta_{je}} = \bar{\Lambda}_j + \Lambda_{ji}\,N_{ji} + \Lambda_{js}\,N_{js} + \Lambda_j\,a_j. \tag{A16}$$

Insert (A7) and (A11) in (A16)

$$K_j = \bar{\Delta}_j + \Delta_j a_j, \tag{A17}$$

where

$$\bar{\Delta}_j\left(z_1,(E_j)\right) \equiv \left(\bar{\Lambda}_j + \Lambda_{js}\,\bar{\theta}_j - \frac{w_j\,a_{je}}{\beta_{je}} + a_{ji}\,\Lambda_{ji}\right)\left(\frac{w_j}{\beta_{je}} + \Lambda_{ji}\right)^{-1}\frac{1}{\bar{\beta}_j\,z_j},$$

$$\Delta_j\left(z_1,(E_j)\right) \equiv \left(\tilde{b}_{ji}\,\Lambda_{ji} + \theta_j\,\Lambda_{js} + \Lambda_j - \frac{w_j\,\tilde{b}_{je}}{\beta_{je}}\right)\left(\frac{w_j}{\beta_{je}} + \Lambda_{ji}\right)^{-1}\frac{1}{\bar{\beta}_j\,z_j}.$$

Substitute (14) into (21)

$$\sum_{j=1}^{J} K_j = \sum_{j=2}^{J} a_j N_j - G. \tag{A18}$$

Substitute (A17) into (A18)

$$\sum_{j=1}^{J} \bar{\Delta}_j + \sum_{j=1}^{J} \Delta_j a_j = \sum_{j=2}^{J} (\Delta_j a_j - a_j N_j) - G. \tag{A19}$$

From (A19) we determine α_1 as:

$$a_1 = \phi(z_1, (E_j), \{a_j\}) \equiv \left(\sum_{j=1}^{J} \bar{\Delta}_j + G + \sum_{j=2}^{J} (\Delta_j - N_j) a_j \right) \frac{1}{N_1 - \Delta_1}. \tag{A20}$$

From $R_g \hat{g}_j = \gamma_j \hat{y}_j$ in (12) and (15) we solve:

$$R_g = \frac{1}{G} \sum_{j=1}^{J} \gamma_j \hat{y}_j \bar{N}_j . \tag{A21}$$

All other variables are solved as functions of and z_1, (E_j), and $\{\alpha_j\}$ by: r and w_j with (A2) → p_j from (A4) → α_1 in (A20) → K_j in (A17) → N_{ji} and N_{je} from (A11) → N_{js} with (A7) → K_{je}, K_{js}, and K_{ji} from (A1) → $\hat{y}_j$ from (A5) → F_{ji}, F_{js} and F_{je} with the definitions → c_j and s_j by (12) → $Y_{je} = w_j N_{je} / \beta_{je}$ → $K = \Sigma_j K_j$ → R_g in (A21) → p_g in (7) → $\bar{g}_j$ in (12). From this procedure, (A19), (5) and (11), we have

$$\dot{a}_1 = \bar{\Phi}_1(z_1, (E_j), \{a_j\}) \equiv \lambda_1 \hat{y}_1 - \phi, \tag{A22}$$

$$\dot{\bar{k}}_j = \Phi_j(z_1, (E_j), \{a_j\}) \equiv \lambda_j \hat{y}_j - \bar{k}_j, j = 2, \ldots, J.$$

$$\dot{E}_j = \Omega_j(z_1, (E_j), \{a_j\}) \equiv \theta_{ji} F_{ji} + \theta_{js} F_{js} + \theta_j C_j - F_{je} - \bar{\theta}_j E_j + \Omega_j\left((E_q) \right). \tag{A23}$$

Taking derivatives of (A19), with respect to t and combining them with (A21), we have

$$\dot{a}_1 = \frac{\partial \phi}{\partial z_1} \dot{z}_1 + \sum_{j=1}^{J} \Omega_j \frac{\partial \phi}{\partial E_j} + \sum_{j=2}^{J} \Phi_j \frac{\partial \phi}{\partial a_j}. \tag{A24}$$

Equal the right-hand sizes of equations (A22) and (A24):

$$\dot{z}_1 = \Phi_1(z_1, (E_j), \{\bar{k}_j\}) \equiv \left[\bar{\Phi}_1 - \sum_{j=1}^{J} \Omega_j \frac{\partial \phi}{\partial E_j} - \sum_{j=2}^{J} \Phi_j \frac{\partial \phi}{\partial \bar{k} a_j} \right] \left(\frac{\partial \phi}{\partial z_1} \right)^{-1}. \tag{A25}$$

In summary, we have proved the lemma.

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